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Math. 7.

# ELEMENTS

OF

# OPTICS.

BY

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10

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# PREFACE.

THE following Elementary Treatise has been compiled, at the desire of the Board of Trinity College, for the use of the junior Students in the University. It is, in a great measure, an abridgment of the Author's *Treatise on Light and Vision;* but with such alterations of form and method as seemed necessary for its present purpose. Experience proves that the same treatise cannot be adapted, with advantage, to the wants both of the general Student, and of the Candidate for Honors. Accordingly, in the selection of the materials for the present volume, which is intended for the former, all but the more essential and elementary portions of the science have been excluded; while, at the same time, the attempt has been made to present these under the simplest form consistent with demonstrative exactness. How far the writer has succeeded in this attempt—one of admitted difficulty—must be left to the experience of the Tutors.

The writer has to express his acknowledgments to the Board, for the assistance afforded by them to the present publication.

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# ELEMENTS OF OPTICS.

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## CHAPTER I.

### OF DIRECT LIGHT.

1. Def. Any body which excites in us the sensation of light is termed *luminous*.

2. The light of all luminous bodies emanates from them in all directions.

This is evident from the fact, that such bodies are visible in whatever position the eye may be with respect to them, if no obstacle be interposed. Thus the light of a candle is seen in all parts of a room, when there is no opaque body between the eye and the flame; and the light of the sun and stars is diffused throughout the celestial spaces.

3. Light is propagated with finite velocity.

This discovery was made by the Danish astronomer, Olaus Roemer, while observing some irregularities in the eclipses of Jupiter's satellites, in the year 1676. It was observed that when Jupiter was in opposition, and therefore nearest to the earth, the eclipses happened *earlier* than they should according to the astronomical tables; while, when Jupiter was in conjunction, and therefore farthest, they happened *later*. The difference between the computed and observed times Roemer justly ascribed to the time taken by the light in traversing the radius of the earth's orbit. This time is found to be $8^m 13^s$; and consequently the velocity of light is such that it traverses 192,500 miles in a second of time.

The observation of the fixed stars is the result of the velocity of their light, combined with that of the earth in its orbit, and its amount depends on the ratio of these velocities. Conversely, the amount of aberration being known by observation, the velocity of light is determined relatively to that of the earth. The velocity thus deduced agrees with that derived from the observation of the eclipses of Jupiter's satellites; thus proving that the direct light of the fixed stars, and the reflected light of the satellites, travel with the same velocity.

4. Light consists of parts, which are separable and independent.

For any portion of light may be stopped by an opaque obstacle, and the rest suffered to pass; and the latter part is found to retain all the properties which it possessed before the separation.

5. Def. The smallest portion of light which can be intercepted, or allowed to pass, is called *a ray*.

6. Def. Whatever affords a passage to the rays of light is called *a medium*.

7. Def. The light which is propagated without interruption, in the same medium, is said to be *direct*.

8. When light encounters the surface of a new medium, it undergoes several modifications, of which the three following are the chief:

I. On meeting the surface of the new medium, the whole, or a portion of the light, is turned back into the medium from which it came. This portion is said to be *reflected*.

II. A portion of the light, in general, enters the new medium, pursuing there an altered course. This portion is said to be *refracted*.

III. A portion of the light which enters the medium is arrested in its progress. This is said to be *absorbed*.

9. In the same homogeneous medium, light is propagated *in right lines.*

The truth of this fundamental law is established by many familiar facts. An object cannot be seen when an opaque body is interposed in the right line connecting it with the eye. Thus if three opaque discs be perforated each with a small aperture, and the eye be placed before the nearest, and the light of a candle behind the farthest, that light will be visible, or not, according as the apertures are, or are not, in the same right line.

The truth of this law may be more obviously evinced by admitting a small beam of the sun's light into a darkened chamber, through an aperture in the window; the particles of dust which float in the air reflecting the beam, and exhibiting its rectilinear form. The same thing is also shown by the shadows of bodies, which are always bounded by right lines.

10. Def. A medium in which no light is absorbed or lost is said to be *perfectly transparent.*

11. In a perfectly transparent medium, the intensity of the light proceeding from a luminous point varies inversely as the square of the distance.

For the intensity of the light received upon any spherical surface whose centre is the luminous point, is as the quantity of the light directly, and inversely as the space over which it is diffused. But none of the light being lost, the quantity of light received upon any spherical surface is the same as that emitted, and is, therefore, constant for a given luminous source. And the space of diffusion, or the area of the spherical surface, is as the square of its radius. Hence the intensity of the light is inversely as the square of the radius, i. e., inversely as the square of the distance.

Let the light be supposed to emanate from the points of an uniformly luminous surface, which we shall suppose to be a small portion of a sphere. Then the quantity of light emitted is proportional to the quantity emitted by a single point, and the number of points (or area) conjointly. Hence if $A$ denote the area of the luminous surface, and $I$ the quantity emitted from a single point, which is a measure of the absolute brightness, the intensity of the illumination, at any distance, $D$, is

$$\frac{AI}{D^2}.$$

12. Required the illumination of a small plane surface, exposed to the light of any source.

A plane surface, whose dimensions are small in comparison with the distance, and which is perpendicular to the incident light, may, without sensible error, be considered as a portion of a spherical surface concentric with the luminary. The degree of illumination, therefore, or the intensity of the light received upon such a plane, is expressed by the formula given in the preceding Article.

When the surface is *inclined* to the incident light, the quantity of the light which it receives is diminished in the ratio of unity to the sine of the angle of inclination. The intensity of the illumination is, therefore, diminished in the same proportion, and is expressed by the formula

$$\frac{AI\sin\theta}{D^2},$$

$\theta$ being the inclination of the surface to the incident light.

13. Experience proves that the eye is incapable of comparing directly two lights, so as to determine their relative intensity. But, although unable to estimate *degrees*, the eye can detect *differences* of intensity with much precision; and with this power it is enabled (by the help of the principles just established) to compare the intensities of two lights *indirectly*.

Let two portions of the same paper (or any similar reflecting surface) be so disposed that one of them shall be illuminated by one of the lights to be compared, and the other by the other, the light being incident upon each at the same angle. Then let the distance of one of the lights be altered, until there is no longer any appreciable difference in the intensity of the illuminated portions. The illuminating powers of the lights will then be as the squares of their respective distances; and their absolute intensities as the illuminating powers directly, and as the luminous surfaces inversely. For, if $I$ and $I'$ denote the absolute intensities of the two lights; $A$ and $A'$ the areas of the luminous surfaces; and $D$ and $D'$ their distances from the paper; then the intensities of illumination are $\frac{AI\sin\theta}{D^2}$, and $\frac{A'I'\sin\theta}{D'^2}$, respectively; and these being rendered equal in the experiment,

$$\frac{AI}{D^2}=\frac{A'I'}{D'^2}.$$

The following simple and convenient mode of practising this method was suggested by Count Rumford. A small opaque cylinder is interposed between the lights to be compared and a screen. In this case it is obvious that each of the lights will cast a shadow, which is illuminated by the other light, while the remainder of the screen is illuminated by both lights together. If, then, one of the lights be moved until the *shadows* appear of equal intensity, their illuminations are equal, and, therefore, the illuminating powers of the two lights to one another as the squares of their distances from the screen.

Ritchie's *photometer* is an instrument founded on the same principle. It consists of a rectangular box, open at the opposite ends, within which are fixed two rectangular pieces of plane looking-glass, with their reflecting surfaces turned outwards, each inclined to the top at an angle of 45°. These surfaces meet in the centre of the top of the box, where there is a narrow slit, covered with oiled paper. In using the instrument, the box is to be placed between the two lights to be compared, and then moved towards one or other until the paper appears equally illuminated in its two halves. The illuminating powers of the two lights, then, are as the squares of their distances from the centre of the box.

## CHAPTER II.

### OF LIGHT REFLECTED AT PLANE SURFACES.

14. WHEN light meets the surface of a new medium, a portion of it is always turned back, or *reflected*.

This reflexion is twofold. Thus when a beam of solar light, which is admitted into a darkened chamber through an aperture in the window, is allowed to fall upon a polished metallic mirror, a reflected beam is seen pursuing a determinate direction after leaving the mirror; and if the eye be placed in this direction, it will perceive a brilliant image of the sun. This beam is said to be *regularly reflected*, and its intensity increases with the polish of the mirror. But it is observed also, that in whatever part of the room the eye is placed, it can always distinguish the portion of the mirror which reflects the light; some of the rays, consequently, are reflected in all directions. This portion of the light is said to be *irregularly reflected*, and its quantity decreases with the polish of the mirror.

Irregular reflexion is due, mainly, to the inequalities of the reflecting surface, which is thus composed of an indefinite number of reflecting surfaces in various positions, and which reflect the light in various directions. The laws of the phenomenon are, therefore, reducible to those of regular reflexion.

15. DEFS. The *angles of incidence* and *reflexion* are the angles which the incident and reflected rays make with the perpendicular to the reflecting surface at the point of incidence. The *angle of deviation* is the angle which the reflected ray contains with the incident ray produced.

Thus (fig. 1) MN being the reflecting surface, IO the incident, and OR the reflected ray, IOP is the angle of incidence; ROP is the angle of reflexion, and ROK is the angle of deviation.

16. The angles of incidence and reflexion are in the same plane, and are equal.

This is the law of reflexion, and is universally true, whatever be the nature of the light itself, or that of the body which reflects it.

17. The truth of this fundamental law is easily established experimentally.

Let a rectangular piece of wood or pasteboard (fig. 2) be provided, having its opposite sides bisected by the lines AB, CD; and let it be immersed perpendicularly in water as far as the line AB, and so placed that a small beam of the sun's light, admitted through an aperture in the window of a darkened room, may be incident at the point O, along the line EO. A portion of this ray will be observed to be turned back, or *reflected*, along the line OF, in the same surface; and the distances CE, CF, being measured, are found equal. Now, since the incident and reflected rays, EO and FO, together with the perpendicular to the reflecting surface, CO, all lie in the plane of the board, the angles of incidence and reflexion are in the same plane. And since the distances CE and CF are equal, and the angles at C right, the angles COE, COF, or the angles of incidence and reflexion, are equal.

The following experimental proof of the truth of the law is capable of greater accuracy.

Let MN (fig. 3) be any plane horizontal reflecting surface, as, for example, the surface of water or mercury at rest; and let SI be a ray of the sun incident upon it, and reflected in the direction IO. Now the centre of a graduated circle being placed at any point, O, of this line, let S′OI and S′OH, the angular distances of the sun from its reflected image, and from the horizon, be measured. It is then found that the former angle is always double of the latter, and consequently S′OH = HOI. But OS′ being parallel to IS, and OH to MN, S′OH = SIN, and HOI = OIM. Hence SIN = OIM, or the incident and reflected rays make equal angles with the reflecting surface.

18. It is an obvious consequence of this law, that if the inci-

dent and reflected rays be given in position, the position of the reflecting surface is determined. Hence if we desire to measure the angle contained by two reflecting surfaces, we have only to make the same ray fall in succession on each, turning the whole system round until the ray reflected from the second surface coincides in direction with that before reflected from the first. The angle through which the system is turned is, then, the supplement of the angle between the surfaces.

This simple principle has been applied in the *reflecting goniometer*, an instrument invented by Dr. Wollaston for the measurement of the angles of crystals.

19. If the reflected ray meet a second reflecting surface, inclined to the first at any angle, the plane of reflexion being the same, the angle which the ray reflected from the second surface makes with the ray incident upon the first, is double the angle contained by the two surfaces.

Let MO, NO (fig. 4), be the sections of the two surfaces made by the plane of reflexion, ABCD the course of the reflected ray. Then, since $CBO = ABM$, $BCN = ABM + MON$; and, since $DCO = BCN$, $CDM = BCN + MON = ABM + 2MON$. But $CDM = ABM + CEA$; therefore $CEA = 2MON$.

When the ray undergoes any number of successive reflexions by the two surfaces, it is shown in the same manner that the angles which it makes with the reflecting surfaces, at each succeeding reflexion, form an arithmetical progression, whose common difference is the angle contained between the two surfaces; and, therefore, that the angle contained by the original ray, and that which has undergone $2n$ reflexions, is equal to $2n$ times the inclination of the surfaces.

The preceding theorem contains the principle of the well-known instrument called Hadley's sextant. In this instrument the ray from one object is reflected successively by two mirrors, and, by altering the inclination of the mirrors, its ultimate direction is made to coincide with that of the ray proceeding from another object seen *directly*. In this case, then, the angle contained by the twice-reflected ray with its original direction is double the inclination of the mirrors. But the twice-reflected ray being made to coincide, in the process of observation, with the direct ray of another object, this angle is the angular distance of the two objects.

20. Defs. A beam of rays, meeting in a common point, is called *a pencil of rays;* and the point in which they meet, or tend to meet, is called the *focus* of the pencil. The rays are said to *diverge* or *converge*, according as they proceed from, or tend to, the focus. When the focus is infinitely distant, the rays are *parallel*.

21. When a pencil of rays, diverging from a point, is incident upon a plane reflecting surface, the reflected rays also diverge from a point; and the two foci are at equal distances from the surface, and at opposite sides.

Q being the focus of the pencil incident upon the reflecting surface ORR′ (fig. 5), let QR be any one of the incident rays which is reflected in the direction RS; and let RS be produced backwards, until it meets the perpendicular QO let fall from the point Q upon the reflecting surface in $q$. Then QRO = SRR′ = $q$RO. Wherefore, in the triangles QRO, $q$RO, the angles at R are equal, the angles at O are right, and the side OR is common. Hence QO = $q$O. In the same manner it may be shown that any other ray of the reflected pencil, R′S′, when produced backwards, will meet the perpendicular QO in $q$. All the reflected rays, therefore, diverge from the point $q$, which is therefore the focus of the reflected pencil.

22. When an *object* is presented to a plane mirror, the rays diverging from each point fall upon the surface, and, after reflexion, diverge from a point as much behind the mirror, as the corresponding point of the object is before it. Thus, for every point of the object, there is a corresponding point behind the reflecting surface from which the rays diverge after reflexion; and if the eye be placed so as to receive these divergent rays, it will see an *image* exactly corresponding to the object in magnitude and position. The only difference is that, as the *faces* of the object and its image are opposed, the position of the object with respect to right and left will be *inverted* in the image.

The image of an object presented to a plane mirror is *virtual*, i. e., the rays never actually meet there. But as the rays meet the eye in the same manner as if they had proceeded from a real object, the mind is at first impressed with the belief of the existence of such an object; and it is only by experience, and the associated sensations of touch, that the illusion is dispelled.

23. When the object is inclined to the mirror, the image will be inclined to it also, and at an equal angle.

Let PQ (fig. 6) be a rectilinear object presented to the mirror MN. Let the line PQ be produced to meet the line MN in O, and at the point O let the line $Op$ be drawn, forming the angle $MOp = MOP$. Then the perpendiculars, $PMp$, $QNq$, being let fall upon the line MN, and produced to meet the line $Op$ in $p$ and $q$, the intercepted portion $pq$ is the image of PQ. For, in the triangles, PMO, $pMO$, the angles at M and O are equal, and the side MO common; so that $pM = PM$, and $p$ is, therefore, the image of P. In the same manner it follows, that any other point of the line $pq$ is the image of the corresponding point of PQ. But PQ and $pq$ are equal, and equally inclined to the reflecting surface.

Hence the angle contained by the object and image is double the inclination of the former to the mirror.

24. When an object is placed between two parallel mirrors, an infinite number of images is formed, increasing in distance, and decreasing in intensity; and these images are all situated in the right line passing through the object and perpendicular to the planes of the two mirrors.

For the image of the object produced by the first mirror is in this right line, and at an equal distance behind the mirror. The rays diverging from this image fall upon the second mirror, and after reflexion diverge from a *second* image in the same right line, at an equal distance behind that mirror. These rays are again incident upon the first mirror; and thus a series of images is produced, the distances of which increase continually. A similar series is produced, in which the first reflexion takes place at the *second* mirror.

25. When an object is placed between two mirrors, inclined at any angle, the images are disposed upon the periphery of a circle, whose centre is upon the line of intersection of the two mirrors, at the foot of the perpendicular let fall from the object upon that line, and whose radius is that perpendicular.

Let CA, CB (fig. 7), be the sections of the two mirrors formed by a perpendicular plane passing through the object Q. Then, if upon the line OA the perpendicular QO be let fall, and produced equally to $q_1$, $q_1$ will be the place of the image of Q formed by the mirror CA. The rays diverging from this image are incident upon the second mirror CB; and the place of the second image is, in like manner, found by letting fall the perpendicular $q_1O'$, and producing it equally to $q_2$; and so on. Now, in the triangles QOC, $q_1OC$, $q_1O = QO$, CO is common, and the angles at O are equal; therefore $Cq_1 = CQ$. In like manner it can be shown that $Cq_2 = Cq_1 = CQ$.

In this case the number of images is limited; for when any one of the images falls upon the portion of the circumference intercepted by the lines AC, BC produced backwards, it is behind *both* mirrors, and there can be no further reflexion.

The forms of the images shown by the *kaleidoscope* may be understood from the principles here laid down.

26. We have hitherto spoken of the *direction* of the reflected light, and seen that it is subject to strict mathematical laws, which are independent of the nature of the reflecting substance. The same thing does not hold of its *intensity*, which is found to vary greatly with the medium. The laws which govern the intensity of reflected light belong to the theory of polarized light, and cannot therefore be introduced here. The following leading facts have been established experimentally.

I. The quantity of light regularly reflected from any surface increases with the angle of incidence, the increase being very slow at moderate incidences, and becoming very rapid at great incidences. Thus water at a perpendicular incidence, according to the experiments of Bouguer, reflects only 18 rays out of 1000; at an incidence of 40° it reflects 22 rays; at 60°, 65 rays; at 80°, 333 rays; and at $89\frac{1}{2}$°, 721 rays.

II. The quantity of light reflected at the same incidence varies both with the nature of the medium upon which the light falls, and with that from which it is incident. Thus, at a perpendicular incidence, the number of rays reflected by water, glass, and mercury, are 18, 25, and 666, respectively, the number of incident rays being 1000. The dependence of the quantity of the reflected light upon the medium from which it is incident is easily shown by immersing a plate of glass in water or oil.

III. The differences in the reflective powers of different substances are much more marked at small than at great incidences. Thus water and mercury—the first of which reflects but the one-fiftieth part of the incident light at a perpendicular incidence, while the latter reflects two-thirds—are equally reflective at an incidence of 89½°, the number of rays reflected at this angle being, in both cases, 721 out of 1000.

# CHAPTER III.

## OF LIGHT REFLECTED AT SPHERICAL SURFACES.

27. When a ray of light is incident upon a curved reflecting surface, it is reflected by it in the same manner as by the plane which touches the surface at the point of incidence.

Of all curved surfaces the spherical is the simplest. It is also, on account of the comparative facility with which it may be wrought, that chiefly used in the construction of optical instruments. On both accounts it demands the chief consideration.

28. Lemma.—If through any point Q (fig. 8) two lines be drawn to meet the surface of a sphere, one of which, QS, passes through the centre, while the other, QR, is inclined to QS at a small angle, the angle RQS, is to the angle at the centre, RCS, in the inverse ratio of the distances of their vertices from the surface.

For, in the triangle QRC, $\frac{\sin \mathrm{RQS}}{\sin \mathrm{RCS}} = \frac{\mathrm{CR}}{\mathrm{QR}}$; but the angles RQS, RCS, being small, are as their sines, *q. p.*; and the point R approaching indefinitely to S, the distances CR and QR are ultimately equal to CS and QS. Hence

$$\frac{\mathrm{RQS}}{\mathrm{RCS}} = \frac{\mathrm{CS}}{\mathrm{QS}}.$$

29. A small pencil of rays being incident perpendicularly upon a concave spherical surface, it is required to find the focus of the reflected pencil.

Let RS (fig. 9) be any section of the sphere, formed by a plane passing through the centre, and through Q, the focus of the incident pencil; the line QC, joining these points, is perpendicular to the surface at S, and is, therefore, the axis of the pencil.

Let QR be any ray of the incident pencil, meeting the surface at R, and reflected in the direction R$q$; and let CR be the radius drawn to the point of incidence. Then, since CR is perpendicular to the tangent plane at the point R, the angles QRC, $q$RC, are the angles of incidence and reflexion, and are therefore equal. Hence,

$$RqS - RCS = RCS - RQS; \text{ and } RQS + RqS = 2RCS.$$

But, if the distances QS and $q$S be denoted by $u$ and $v$, RQS $= \frac{r}{u}$ RCS, R$q$S $= \frac{r}{v}$ RCS (28), and substituting and dividing by $r$, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}.$$

Now, the distance $u$ being the same for all the rays of the incident pencil, the distance $v$ will be (approximately) the same for all the rays of the reflected pencil; and all these rays will converge, very nearly, to the same point.

The point thus determined is sometimes called the *geometric focus* of the pencil; and it may be shown that, whatever be the breadth of the pencil, the number of rays collected near it is greater than at any other point, and therefore the density of the light a maximum.

This formula includes the case of reflexion at a *plane* reflecting surface, already given (21); for, in that case, $\frac{1}{r} = 0$, and consequently,

$$u + v = 0.$$

The radiant and its conjugate, therefore, are at equal distances from the surface, and at opposite sides.

30. The distances of the foci of incident and reflected rays from the surface are to one another as their distances from the centre.

For the preceding formula may be put under the form $\frac{1}{v} - \frac{1}{r} = \frac{1}{r} - \frac{1}{u}$; from which we have

$$\frac{r - v}{v} = \frac{u - r}{u};$$

in which $u - r$ and $r - v$ are the distances of the foci from the centre.

31. In the case of *converging* rays (fig. 10),

$$\text{R}q\text{S} - \text{RCS} = \text{RCS} + \text{RQS}; \text{ and } \text{R}q\text{S} - \text{RQS} = 2\text{RCS}.$$

Hence, as before,

$$\frac{1}{v} - \frac{1}{u} = \frac{2}{r};$$

a formula which differs from the preceding only in the sign of $u$. Now it will be remarked, that in the case of *diverging* rays (fig. 9), the distance of the radiant from the surface, QS, is measured *towards the incident light*, or *in front* of the mirror; while in that of *converging* rays (fig. 10) it is in the opposite direction. If, therefore, the distance be considered as *positive* in the former case, it must be *negative* in the latter; so that the latter formula is really included in the former.

32. When the incident rays are *parallel*, $u$ is infinite, and $\frac{1}{u} = 0$; and if the value of $v$, in this case, be denoted by $f$, the formula of (25) becomes $\frac{1}{f} = \frac{2}{r}$, or

$$f = \tfrac{1}{2}r.$$

The focus of the reflected rays, in this case, is called the *principal focus;* and its distance from the surface the *principal focal distance*, or simply the *focal length* of the reflector. The focal length of a spherical reflector is then equal to half the radius.

33. Substituting $f$ for $\frac{1}{2}r$ in the formula (29) it becomes

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

It follows from this formula, that the foci of the incident and reflected rays are *reciprocal;* i. e., that if $q$ be made the radiant, Q will be its conjugate. For the formula is unaltered if $u$ and $v$ be transposed.

Solving the equation for $v$,

$$v = \frac{uf}{u-f};$$

from which we learn that "the distance of the radiant from the principal focus is to the focal length, as the distance of the radiant from the surface to the distance of the conjugate from the same."

Subtracting $f$ from both members of the latter formula,

$$v - f = \frac{uf}{u-f} - f = \frac{f^2}{u-f}; \text{ or}$$

$$(u-f)(v-f) = f^2.$$

That is, "the distance of the centre from the principal focus is a mean proportional between the distances of the radiant and conjugate from the same."

34. The corresponding positions of the radiant and conjugate are inferred from the formulæ of (29 or 33).

When the radiant is infinitely distant, or $\frac{1}{u} = 0$, $v = f$, or the conjugate coincides with the principal focus.

As the radiant approaches the surface from an infinite distance, its conjugate recedes from it. For when $u$ decreases, $v$ increases.

When the radiant reaches the centre, or $u = r$, $v = r$, and the conjugate meets it at the same point.

When the radiant comes between the centre and the surface, or $u < r$, $v > r$, or the conjugate has moved beyond the centre.

When the radiant reaches the principal focus, or $u = f$, $\frac{1}{v} = 0$; or the distance of the conjugate becomes infinite, and the reflected rays are parallel.

When the radiant is between the principal focus and the surface, or $u < f$, $v$ becomes *negative*. To understand the meaning of this change of sign, it must be observed, that as the positive values of the distances $u$ and $v$ have been measured from the surfaces *towards the incident light*, or *before* the mirror, the negative values are to be taken in an opposite direction. The negative value of $v$, therefore, indicates that the focus of the reflected rays is *behind the mirror*, and consequently that the reflected rays *diverge*, instead of converging as before. It is evident that the rays do not actually meet there; and that the focus of the reflected pencil is, in this case, a *virtual* focus, as in the case of reflexion at plane surfaces.

As the radiant approaches the surface from the principal focus, its conjugate approaches it likewise in an opposite direction; until when $u = 0$, or the radiant reaches the surface, $v = 0$, and its conjugate meets it there.

Finally, when the radiant crosses to the other side of the surface, or the incident rays *converge*, its conjugate recedes from the surface; and as the radiant moves from the surface to an infinite distance, the conjugate moves from it to the principal focus.

These various consequences of the formula may be summed up in the following rule:—"The radiant and its conjugate move in opposite directions, and meet at the centre and surface of the mirror."

35. When the reflecting surface is *convex*, and the incident rays *diverge*, the angle of incidence (fig. 11) is equal to $RCS + RQS$, so that

$$RqS - RQS = 2RCS,$$

as in the case of *converging* rays incident upon a *concave* mirror (31); and the resulting formula is consequently the same.

When the incident rays *converge*, the angle of incidence (fig. 12) is equal to $RCS - RQS$, so that

$$RqS + RQS = 2RCS;$$

as in the case of *diverging* rays incident upon a *concave* mirror.

The reason of this correspondence will appear from the circumstance, that, in the concave mirror, the radius is measured from the surface *towards* the incident light, while in the convex it is measured in the opposite direction, and must be considered as having changed sign. Now it is obvious that a change of sign of $r$, in the formula connecting $u$, $v$, and $r$, is equivalent to a simultaneous change of the signs of $u$ and $v$; and this, by the rule of (31), is equivalent to a change of divergent rays into convergent, and *vice versâ*. Hence all that has been said of the concave mirror applies directly to the convex, substituting divergence for convergence, and *vice versâ*.

36. When an *object* is presented to a spherical reflector, the rays diverging from its several points will diverge from, or converge to, corresponding points after reflexion; and the aggregate of these foci of reflected pencils constitutes the optical *image* of the object.

Hence, to determine the form and position of the image of any object, we must determine the foci conjugate to its several points. This is done by drawing (fig. 13) from each point of the object PQ, lines PC, QC, through the centre of the spherical surface, and computing on these lines (which are the axes of the corresponding pencils), the place of the foci of the reflected pencils, $p$, $q$, according to the formula already given.

Accordingly, the *relative distances* of the object and image are determined by the same rules as those of the conjugate foci. The object and image, therefore, lie always on the same side of the principal focus; they move in opposite directions, and meet at the centre and surface of the mirror.

37. As the axes of the several pencils cross at the centre of the spherical surface, it is evident that the image will be *inverted* with respect to the object, when they lie at opposite sides of the centre; *erect*, when at the same side. From this it follows that the image of an object presented to a convex mirror is always erect, while that produced by a concave mirror is erect only when the object is between the principal focus and the surface. It is also evident that the image is *real* when inverted, and when erect *imaginary*.

38. As the axes of the pencils diverging from the extreme points of the object intersect at the centre of the sphere, it follows that the *linear magnitudes* of the object and image are to one another as their distances from the centre; or, which is the same thing (30), as their distances from the surface. Hence if the linear

magnitudes of the object and image be denoted by $a$ and $a'$, there is

$$\frac{a'}{a} = \frac{v}{u} = \frac{f}{u-f}.$$

39. It follows from the preceding, that if a spectator stands in front of a concave mirror, and between the principal focus and the surface, he will see an *erect* image of his face behind the mirror; and this image, being more remote from the centre than the object, will be *magnified* with respect to it. But if the spectator recedes from the mirror until he is beyond the centre, he will then see an *inverted* image of his face in front of the mirror, and at the other side of the centre; and this image, being nearer to the surface than the object, will be *less* than it.

The image in the latter case is sometimes called an *aerial* image. The effect of such an image is much more striking when it is produced by another object. Thus, if any object be placed before the mirror, and at a short distance from the centre on either side, an inverted image of it will be formed at the other side of the centre; and the rays diverging from the points of this image, and reaching the eye, will appear to emanate from a real object. The illusion is heightened by inclining the mirror a little, so that the object may be at one side of its axis, and the image, therefore, at the other side. By such means the object may be concealed from the spectator, and its aerial image alone visible.

The image produced by reflexion at a *convex* mirror is always nearer to its centre than the object, and is consequently always *less* than it. Such a mirror is sometimes used in ornamental furniture, to give a miniature representation of the parts of a room.

40. The quantity of light in the image of a small luminous object, formed by a concave speculum, varies as the square of the aperture of the speculum directly, and inversely as the square of the distance of the object from it.

The angle subtended by the aperture of the speculum at the object being small, the surface of the speculum may be considered as a portion of the hemisphere whose centre is the object, and radius its distance from the speculum. The quantity of light incident upon it, therefore, will be to that incident upon the entire hemisphere, $Q$, in the ratio of their areas, i. e., as $\pi A^2 : 2\pi u^2$; $A$ being the semi-aperture of the speculum, and $u$ the distance of the object. Hence the quantity of light incident upon the speculum is $Q\frac{A^2}{2u^2}$. But the quantity of light reflected is to that incident in a constant ratio, and is, therefore, equal to $pQ\frac{A^2}{2u^2}$, $p$ denoting the constant ratio. This is the quantity of the light in the image, no light being supposed to be lost by absorption; hence, if it be denoted by $Q'$,

$$\frac{Q'}{Q} = \tfrac{1}{2} p \, \frac{A^2}{u^2}.$$

41. The brightness of the image varies as the square of the aperture of the speculum directly, and inversely as the square of the distance of the image from it.

For the brightnesses, or the intensities of the light, in the object and image, are as the quantities of light directly, and inversely as their areas, or the squares of their linear magnitudes; that is, $\frac{I'}{I} = \frac{Q'}{Q} \cdot \frac{a^2}{a'^2}$, $I$ and $I'$ denoting the intensities. And substituting for $\frac{Q'}{Q}$ and $\frac{a}{a'}$, their values (40), (38),

$$\frac{I'}{I} = \tfrac{1}{2} p \, \frac{A^2}{v^2}.$$

It follows from the preceding, that the intensity of the light in the image of the sun formed by a concave speculum, varies as the square of the aperture directly, and inversely as the square of the focal length. But the heat which accompanies light of a given kind is, in general, proportional to the intensity of the latter; so that if the aperture of the speculum be considerable, and its focal length small, the heat in the solar image will be so great as to burn combustible substances with ease, and even to fuse the most refractory bodies. By the aid of a mirror whose aperture was four feet, and focal length thirty-eight inches, made by M. Villele of Lyons, silver was fused in seven seconds and a half, cast iron in sixteen seconds, and a piece of granite in less than a minute.

42. In computing the place of the focus of the reflected pencil, it has been assumed that the angles formed by the incident and reflected rays with the axis were indefinitely small, and the focus of reflected rays in this case has been called the geometric focus of the pencil. When, however, the incident ray is inclined to the axis at any finite angle, the reflected ray will not meet the axis in this point; and the deviation will be greater, the greater the inclination. The distance of the intersection of the extreme ray with the axis, from the geometric focus, is called the *aberration* of the extreme ray.

Hence, the rays diverging from any point in an object presented to a spherical mirror do not meet accurately in a point after reflexion. The image of each point, therefore, is a small surface, and the overlapping of these surfaces will produce confusion in the entire image. To avoid this evil, the specula of the larger telescopes are ground into the form of the paraboloid of revolution, this surface possessing the property of reflecting accurately to a point all the rays which are incident upon it parallel to its axis.

The parabolic mirror is also employed in lighthouses. The lamp being placed in the focus of the parabola (which is also the focus of the mirror) the reflected rays will emerge parallel to the axis; so that no light is lost by diffusion.

# CHAPTER IV.

## OF LIGHT REFRACTED AT PLANE SURFACES.

43. WHEN light is incident upon the surface of a transparent medium, a portion, in general, enters the medium, pursuing there an altered direction. This portion is said to be *refracted;* and the phenomenon itself is called *refraction*.

DEFS. The angles which the incident and refracted rays make with the perpendicular to the surface at the point of incidence are called the *angles of incidence* and *refraction;* and the angle contained by the refracted ray with the incident ray produced is called the *angle of deviation*.

When the ray passes from a rarer into a denser medium, the angle of incidence is, in general, greater than the angle of refraction, and the deviation takes place *towards the perpendicular* to the bounding surface. On the contrary, when the ray passes from a denser into a rarer medium, the angle of incidence is less than the angle of refraction, and the deviation is *from the perpendicular*.

44. The angles of incidence and refraction are in the same plane; and their sines are in an invariable ratio.

This is the fundamental law of refracted light, the truth of which is established by experiment. In order to verify it, it is only necessary to measure several angles of incidence at the surface of the same medium, and the corresponding angles of refraction. The following mode of performing this measurement was devised by Newton.

Let a straight beam be provided, having two plates, attached near the extremities perpendicular to the line of the beam, and therefore parallel. The upper plate being perforated in any point, a point is to be marked on the lower, corresponding to it in position. Then a glass vessel, whose bottom is perfectly plane, being placed on the upper plate, and partly filled with water, let a ray of the sun's light be incident on the surface of the fluid; and let the position of the beam be shifted, until the refracted ray, passing through the aperture in the upper plate, shall meet the lower in the corresponding point. At the same moment, let the altitude of the sun be taken, and the inclination of the beam to the horizon measured.

Now, the surface of the fluid being horizontal, the altitude of the sun is equal to the angle which the incident ray makes with the surface, or to the complement of the angle of incidence. Also, the refracted ray being evidently parallel to the beam, the inclination of the beam to the horizon is equal to the angle which the refracted ray makes with the surface, or to the complement of the angle of refraction. Thus, the experiment being repeated at different altitudes of the sun, we obtain several angles of incidence with their corresponding angles of refraction; and their sines, being taken from the trigonometrical tables, will be found always in the same ratio.

45. If $\phi$ and $\psi$ be employed to denote the angles which the portions of the ray in the rarer and denser medium, respectively, make with the perpendicular to the common surface, the second part of the law of refraction is expressed by the equation,

$$\sin \phi = \mu \sin \psi,$$

$\mu$ being a constant quantity.

This constant is termed the *index of refraction;* and since $\phi > \psi$, it is always greater than unity.

When a ray of light passes into any medium from a vacuum, the index of refraction is in that case termed the *absolute index* of the medium. For air, and the gases, it exceeds unity by a very small fraction; for water, $\mu = 1.336$; for crown glass, $\mu = 1.535$; for diamond, $\mu = 2.487$; and, for chromate of lead, $\mu = 3$.

46. When the angles of incidence and refraction are very

small, they are, very nearly, proportional to their sines. The preceding formula becomes, therefore, in this case,

$$\phi = \mu\psi;$$

whence

$$\phi - \psi = (\mu - 1)\,\psi = \frac{\mu - 1}{\mu}\,\phi.$$

Accordingly, the deviation of a ray, which passes nearly perpendicularly through a refracting surface, is *q. p.* proportional to the angle of incidence.

47. The angle $\psi$, which the portion of the ray in the denser medium makes with the perpendicular, can never exceed a certain limit.

For it is evident that $\sin\psi$, and therefore $\psi$ itself, is greatest, when $\sin\phi$ is so. But the greatest value of $\sin\phi$ is unity, to which it is equal when $\phi = 90°$. Wherefore if $\Psi$ denote the corresponding value of $\psi$,

$$\sin\Psi = \frac{1}{\mu}.$$

Thus, at the surface of water, $\Psi = 48\frac{1}{2}°$, and at the surface of glass, $\Psi = 40\frac{1}{2}°$. But, when the course of the ray is from the denser into the rarer medium, $\Psi$ is the angle of incidence. Hence it follows that no ray can pass from a denser into a rarer medium, when the angle of incidence exceeds this limit.

It is found by experiment that at greater incidences the ray is turned back, or *reflected*, into the denser medium; and as no portion of the light can penetrate the surface, this reflexion is *total.* This is the only case in which light is reflected without a diminution of intensity.

48. When a pencil of rays, diverging from a point, is incident upon a plane refracting surface, the distance of the focus of refracted rays from the surface is to the distance of the focus of incident rays from the same, in the ratio of the sine of incidence to the sine of refraction.

Let Q be the radiant point (fig. 14), and QO the perpendicular from it on the refracting surface. Let QR be any ray of the incident pencil, which is refracted in the direction RS; and let the latter line be produced backwards to meet the perpendicular in $q$. Then in the triangle $\mathrm{QR}q$,

$$\frac{\mathrm{R}q}{\mathrm{RQ}} = \frac{\sin \mathrm{RQ}q}{\sin \mathrm{R}q\mathrm{Q}} = \frac{\sin \mathrm{RQO}}{\sin \mathrm{R}q\mathrm{O}} = \frac{\sin(\mathrm{Inc.})}{\sin(\mathrm{Ref.})}$$

But, when the incidence is perpendicular, the point R coincides with O, and the distances RQ and $\mathrm{R}q$ with OQ and $\mathrm{O}q$. Wherefore,

$$\frac{q\mathrm{O}}{\mathrm{QO}} = \frac{\sin(\mathrm{Inc.})}{\sin(\mathrm{Ref.})}.$$

When the radiant point is in the rarer medium, $\frac{\sin(\mathrm{Inc.})}{\sin(\mathrm{Ref.})} = \mu$, and $q\mathrm{O} > \mathrm{QO}$. When the radiant is in the denser medium, $\frac{\sin(\mathrm{Inc.})}{\sin(\mathrm{Ref.})} = \frac{1}{\mu}$, and $q\mathrm{O} < \mathrm{QO}$. The two cases are comprised in the same formula, if we denote by $u$ and $v$ the distances of the foci of the rays in the rarer, and in the denser medium, respectively; and we have

$$v = \mu u.$$

49. When an *object* is presented to a refracting surface, the rays proceeding from each point will diverge after refraction from a corresponding point, whose distance from the surface is determined by the preceding theorem. The aggregate of these foci of refracted pencils forms the *image* of the object.

The distance of the several points of the image from the surface being greater, or less, than those of the corresponding points of the object, in a constant ratio, it is manifest that the *figure* of the image is *similar* to that of the object; but its *position* and *magnitude* are, in general, different.

When the object is *parallel* to the refracting surface, its image is also parallel to it, at a distance greater, or less, in the ratio of the sine of incidence to the sine of refraction; and the magnitudes of the object and image are equal. It is thus that the bottom of a lake appears nearer than it really is, by the refraction of the surface.

When the object is *inclined* to the refracting surface, its image is likewise inclined to it; and the tangent of the angle of inclination of the image is to the tangent of the angle of inclination of the object, as the sine of incidence to the sine of refraction.

For AB (fig. 15) being the object, and $ab$ the image formed by refraction at the surface RS,

$$\frac{a\mathrm{R}}{\mathrm{AR}} = \frac{\sin \text{Inc.}}{\sin \text{Ref.}} = \frac{b\mathrm{S}}{\mathrm{BS}}.$$

Hence it follows that the lines AB, $ab$, produced, meet the surface RS in the same point O; and

$$\frac{\tan b\mathrm{OS}}{\tan \mathrm{BOS}} = \frac{b\mathrm{S}}{\mathrm{BS}} = \frac{\sin \text{Inc.}}{\sin \text{Ref.}}$$

Hence the reason why a stick appears bent when partly immersed in water, the image of the immersed part forming a smaller angle with the surface than the stick itself, and thus appearing inclined to the part which is extant.

50. When a ray of light traverses a medium bounded by parallel planes, and re-enters the original medium, the emergent ray is parallel to the incident.

Let QRR′Q′ (fig. 16) be the course of the ray incident upon the first surface of the medium at R, and emergent at the second at R′; and $op$, $o'p'$, the perpendiculars to the surfaces at these points. Then, by the law of refraction,

$$\frac{\sin \mathrm{QR}o}{\sin \mathrm{R'R}p} = \frac{\sin \mathrm{Q'R'}o'}{\sin \mathrm{RR'}p'}.$$

But, on account of the parallelism of the surfaces, $\mathrm{R'R}p = \mathrm{RR'}p'$, therefore $\mathrm{QR}o = \mathrm{Q'R'}o'$; and, the angles $\mathrm{QR}o$, $\mathrm{Q'R'}o'$, being in the same plane, QR and Q′R′ are parallel.

51. When a ray of light, passing through the second medium, enters a third which is different from either of the former, the surfaces bounding the media being parallel, the course of the ray in the third medium will be the same as if the light had entered it directly from the first, and is therefore altogether independent of the intermediate medium.

For the demonstration of this law, which may be considered as the foundation of all that relates to successive refraction by different media, we must resort to experience. It is proved by experiment that if a ray of light, QRSTV (fig. 17), passing from the medium A, traverses two others, B and C, and finally emerges into the original medium, all the bounding surfaces being parallel, the emergent ray TV is parallel to the incident QR.

Now, if we suppose another ray, Q′S′T′V′, to be incident directly from the first upon the third medium, in a direction parallel to QR, and traversing the third medium, to re-enter the first, it follows from the last article that the incident and emergent rays, Q′S′ and T′V′, are parallel. Hence, the incident rays, QR and Q′S′, being supposed parallel, the emergent rays, TV and T′V′, are also parallel; and, as they are both equally refracted at the common surface of the same media, ST and S′T′ are likewise parallel.

Hence, when a ray of light passes from any medium, through an intermediate medium, into a third, all being bounded by parallel surfaces, the total deviation of the ray is the same as if it had passed directly from the first into the third.

It is easy to see that this principle may be generalized; and that, whatever be the number of media, the course of the ray in the last will be the same as if the light had been incident directly from the first into the last, parallel to its original direction.

52. The *relative* index of refraction between any two media is equal to the quotient arising from the division of the absolute index of the second by that of the first.

For, since Q′S′ is parallel to QR (fig. 17), and S′T′ to ST, $Q'S'o' = QRm$, and $T'S'p' = TSp$. Hence

$$\frac{\sin Q'S'o'}{\sin T'S'p'} = \frac{\sin QRm}{\sin TSp} = \frac{\sin QRm}{\sin SRn} \times \frac{\sin RSo}{\sin TSp}.$$

But, if the medium A be a vacuum, $\frac{\sin QRm}{\sin SRn}$, $\frac{\sin Q'S'o'}{\sin T'S'p'}$, are, respectively, the refractive indices of the media B and C; and, denoting them by $\mu$ and $\mu'$, we have

$$\frac{\sin RSo}{\sin TSp} = \frac{\mu'}{\mu}.$$

Thus, if it be required to find the relative index of refraction from water into glass, we have $\mu = \frac{4}{3}$, and $\mu' = \frac{3}{2}$, nearly. Hence the relative index is equal to $\frac{9}{8}$, nearly.

53. When a small pencil of rays diverging from a point is incident perpendicularly upon a medium bounded by parallel planes, and emerges into the original medium, it is required to find the focus of the emergent pencil.

Let $u$ and $u'$ denote the distances of the foci of incident and emergent rays from the first surface; and $v$ the distance of the conjugate focus from the same after the first refraction. Then, the thickness of the medium being denoted by $t$, the distances of the conjugate foci from the refracting surface, at the second refraction, will be $v + t$, and $u' + t$; and, the medium being supposed denser than that by which it is surrounded, we have (48),

$$v = \mu u, \quad v + t = \mu(u' + t).$$

Substituting in the latter the value of $v$ given by the former, and reducing, there is

$$u - u' = \frac{\mu - 1}{\mu} t.$$

Accordingly the focus of the pencil is brought nearer to the medium by the double refraction, the distance between the foci of the incident and emergent pencils being to the thickness of the medium in a constant ratio. In the case of glass, the interval is one-third of the thickness of the medium, very nearly.

When an object is presented to a denser medium bounded by parallel planes, the points of the image are nearer the surface, and the difference is constant. Hence the image is parallel to the object, and its form and magnitude are unaltered; the only effect of the refraction being to alter the distance.

54. Defs. Any medium bounded by two plane surfaces which are inclined to one another is called, in optics, a *prism*. The two surfaces are called the *faces* of the prism; the line in which they meet, the *edge*; and the angle which they form, the *refract-*

*ing angle.* Any section of the prism, made by a plane perpendicular to the edge, is called a *principal section.*

We shall limit our attention, in what follows, to the case in which the light is incident in the plane of a principal section, in which case, it is evident, it must continue in that plane.

55. When a ray of light traverses a prism which is denser than the surrounding medium, the total deviation is, in all cases, from the refracting angle.

Let BAC (figs. 18, 19, 20) represent a principal section of the prism; QRR′Q′ the course of the refracted ray, which is in the plane of that section; and let *mn*, *m′n′*, be the perpendiculars to the faces of the prism at the points of incidence and emergence. The angles ARR′, AR′R, may be both acute, one right, or one obtuse.

I. In the first case, the incident and emergent rays, QR, R′Q′ (fig. 18), lie on the sides of the perpendiculars *from the vertex;* and since the portion of the ray in the rarer medium contains a greater angle with the perpendicular than that in the denser, the deviation, both at ingress and egress, is from the vertex of the prism.

II. In the second case, in which one of the angles, ARR′ (fig. 19) is right, and the other acute, there is no deviation at R; and at R′ the deviation is, for the reason already given, from the vertex.

III. In the third case, in which one of the angles ARR′ (fig. 20) is obtuse, and the other acute, since the ray RQ must lie at the side of the perpendicular towards the vertex, and R′Q′ at the opposite side, the deviation at R will be towards the vertex, and at R′ from it. But the angle which the ray makes with the perpendicular to the surface at R′ is greater than at R, the former being the exterior, the latter the interior angle of the triangle RR′*n*. Hence the deviation at R′ is greater than at R, and, therefore, the total deviation, which in this case is the difference of the partial deviations, is from the vertex.

56. The vertical angle of the prism is equal to the sum, or difference, of the angles which the portion of the ray within it

makes with the perpendiculars to the faces at the points of incidence and emergence.

Letting fall the perpendicular, AO, from the point A upon the line RR′ (fig. 18), it is manifest that $n\mathrm{RR}' = \mathrm{RAO}$, $n'\mathrm{R'R} = \mathrm{R'AO}$. But the vertical angle of the prism is the sum, or difference, of the angles RAO, R′AO, according as the angles ARR′, AR′R, are both acute, or one obtuse. Wherefore, denoting this angle by $a$, and the angles which the line RR′ makes with the perpendiculars to the two faces by $\psi$ and $\psi'$,

$$\psi \pm \psi' = a.$$

Hence it follows that the greatest angle of a prism, which can transmit a ray of light, is $2\Psi$. For it is evident that $a$ is greatest when it is equal to the *sum* of the angles $\psi$ and $\psi'$, and that each of these angles has its maximum value (47). For glass, $\Psi = 40°\frac{1}{2}$; hence a glass prism cannot transmit a single ray, if its refracting angle is greater than 81°.

57. It is required to determine the deviation of a ray of light produced by traversing a prism.

The total deviation of the refracted ray is the sum of the deviations at incidence and emergence. Wherefore if $\phi$ and $\phi'$ denote the angles which the incident and emergent rays make with the perpendiculars to the faces at the points of incidence and emergence, the deviations at incidence and emergence are, respectively, $\phi - \psi$, and $\phi' - \psi'$; wherefore, adding and observing that $\psi + \psi' = a$, the total deviation, $\delta$, is

$$\delta = \phi + \phi' - a.$$

The angle of the prism and its position with respect to the incident ray being known, $a$ and $\phi$ are given, and it only remains to determine $\phi'$, in order that the value of $\delta$ may be determined. But if $\mu$ denote the index of refraction of the substance of which the prism is composed,

$$\sin\phi = \mu \sin\psi, \ \sin\phi' = \mu \sin\psi';$$

and combining these with the equation $\psi + \psi' = a$, the angle $\phi'$ may be deduced in terms of $a$ and $\phi$, by elimination.

58. When a ray of light is incident nearly perpendicularly upon a thin prism, the total deviation is constant, and bears an invariable ratio to the angle of the prism.

For in this case the angles of incidence and refraction, being small, are proportional to their sines, so that

$$\phi = \mu\psi, \quad \phi' = \mu\psi'; \quad \text{and } \phi + \phi' = \mu(\psi + \psi') = \mu a.$$

Hence

$$\delta = (\mu - 1)\, a.$$

59. Required the deviation produced by a prism of any refracting angle, when the angles of incidence and emergence are equal.

We have seen that, generally,

$$\phi + \phi' = a + \delta, \quad \psi + \psi' = a.$$

But since, in this case, $\phi = \phi'$, there is also $\psi = \psi'$; and consequently

$$\phi = \tfrac{1}{2}(a + \delta), \; \psi = \tfrac{1}{2}a.$$

Hence we have

$$\sin \tfrac{1}{2}(a + \delta) = \mu \sin \tfrac{1}{2}a;$$

from which $\frac{1}{2}(a + \delta)$, and therefore $\delta$, is determined.

It may be shown that the angle of deviation, in this case, is the least possible; and accordingly if the prism be turned slowly round its axis, the inclination of the emergent to the incident ray will first decrease, and afterwards increase, appearing for a moment to be stationary between the opposite changes. By this principle we are enabled to place a prism, experimentally, in the position in which the refractions are equal at both sides.

60. Required to determine the refractive index of a transparent solid experimentally.

The first step of this process is to polish two plane faces, inclined to one another at a sufficient angle, and to measure that angle by a goniometer. This being done, the prism is to be placed, with its refracting edge vertical, before the object-glass of the telescope of a theodolite, so as to refract along its axis the rays proceeding from a distant mark. The prism is to be turned slowly round its axis, until the deviation is a minimum, and the horizontal circle then read. The prism being then removed, the telescope is to be turned directly to the distant mark, and the reading repeated; the difference of the two readings is the deviation. The angle of the prism and the deviation being obtained, the refractive index is given by the formula,

$$\mu = \frac{\sin \frac{1}{2} (\alpha + \delta)}{\sin \frac{1}{2} \alpha}.$$

To determine the refractive index of a fluid, we have only to enclose it in a hollow prism, whose sides are formed of glass plates with parallel surfaces. For the course of the ray, and the total deviation, will be the same as if it had been incident directly from the air into the fluid, and had emerged similarly, without passing through the glass (51).

61. When a ray of light which has entered a prism falls upon the second surface at an incidence greater than the limiting angle, $\Psi$, it will undergo total reflexion there.

The most important case of total reflexion is that which takes place at the base of a right-angled isosceles prism. When a ray of light is transmitted perpendicularly through one of the sides of such a prism, it falls upon the base at an angle of 45°; and this being greater than the limiting angle, it is totally reflected there. The angle of reflexion being equal to the angle of incidence, the reflected ray will meet the remaining side perpendicularly, and will therefore be transmitted through it, as through the first side, without any deviation. Hence the course of the ray is the same as if it had encountered only a reflecting surface. The *intensity* of the light, however, is very different. The most highly reflective of all known substances reflect little more than $\frac{7}{10}$ths of the incident light at an angle of 45°; whereas, in this case, the reflexion is *total*, and the only light lost is that which is absorbed in its passage through the glass. The reflexion is accordingly more brilliant than that produced by any other means.

62. If a right-angled isosceles prism be placed with one side horizontal and uppermost, and the other be turned towards a dis-

tant object, the rays proceeding from that object, and received perpendicularly upon the latter side, will emerge (as has been shown) perpendicularly through the former, having undergone total reflexion at the base. Thus the object, whose direction is horizontal with respect to the spectator, may be seen by looking directly downwards; and its image may be made to fall upon a piece of paper placed horizontally below the prism, and be traced there with a pencil.

This is the principle of the *camera lucida*, an instrument of great use to the draftsman. In the arrangement described, however, the image is *inverted*, as seen by a spectator having his face towards the object. To remèdy this objection, Dr. Wollaston, the inventor of the instrument, substituted a quadrilateral prism for the triangular one just mentioned. In this prism (fig. 21) the angle at A is right; the opposite angle, D, is a right angle and a half, or 135°; and the remaining angles, B and C, are each three-fourths of a right angle, or 67½°. In this arrangement, it will readily appear, the rays which are incident perpendicularly upon the side AC will undergo total reflexion at CD and DB, and finally emerge perpendicularly at the side AB; and the image will be *erect* with respect to a spectator having his face turned towards the object, the rays proceeding from the upper part of the object going to the upper part of the image, and *vice versâ*.

# CHAPTER V.

## OF LIGHT REFRACTED AT SPHERICAL SURFACES.

63. A small pencil of rays being incident perpendicularly upon a spherical surface bounding a denser medium, required the focus of the refracted pencil.

Let Q (fig. 22) be the focus of the incident pencil; and QCS the line connecting it with the centre, which is, therefore, the axis of the pencil. Let QA be any other ray of the incident pencil; AS the section of the surface containing it and the axis; and A$q$ the refracted ray, or the refracted ray produced, meeting the axis in $q$. Then, the angles of incidence and refraction, QAC, $q$AC, being small by hypothesis, are to one another as their sines, very nearly; and, consequently,

$$\mathrm{QAC} = \mu \times q\mathrm{AC},$$

$\mu$ being the index of refraction. But $\mathrm{QAC} = \mathrm{ACS} - \mathrm{AQS}$, $q\mathrm{AC} = \mathrm{ACS} - \mathrm{A}q\mathrm{S}$; wherefore, substituting and transposing,

$$\mu \times \mathrm{A}q\mathrm{S} = \mathrm{AQS} + (\mu - 1)\,\mathrm{ACS}.$$

But, if the distances, QS, $q$S, be denoted by $u$ and $v$, and the radius CS by $r$, there is (28) $\mathrm{AQS} = \frac{r}{u}\,\mathrm{ACS}$, $\mathrm{A}q\mathrm{S} = \frac{r}{v}\,\mathrm{ACS}$. Wherefore substituting, and dividing by $r$,

$$\frac{\mu}{v} = \frac{1}{u} + \frac{\mu - 1}{r}.$$

In the preceding deduction the refracting surface has been assumed concave. When it is *convex* (fig. 23),

$$\mathrm{QAC}' = \mu \times q\mathrm{AC}',$$

AC′ being the radius produced. But $\mathrm{QAC}' = \mathrm{ACS} + \mathrm{AQS}$, $q\mathrm{AC}' = \mathrm{ACS} + \mathrm{A}q\mathrm{S}$; so that

$$\mu \times \mathrm{A}q\mathrm{S} = \mathrm{AQS} - (\mu - 1)\,\mathrm{ACS},$$

and the resulting formula will differ from that already obtained merely in the sign of $r$. But the radius being measured, in this case, in an opposite direction to that in the former, it is evident that the formula above written includes both cases, if we agree to consider the distances as *positive*, when measured from the surface *towards the incident light*, and negative, when measured in the opposite direction.

64. When the light passes from the denser into the rarer medium, $q\mathrm{AC} = \mu \times \mathrm{QAC}$; and the formula becomes

$$\frac{\mu}{u} = \frac{1}{v} + \frac{\mu - 1}{r},$$

which differs from that of (63) simply in the substitution of $u$ for $v$, and *vice versâ*. The formula of (63), therefore, includes the two cases, if we take $u$ as the focal distance of the rays in the rarer medium, and $v$ that of those in the denser.

65. When the rays, incident upon the *denser* medium, are *parallel*, $\frac{1}{u} = 0$; and, if the corresponding value of $v$ be denoted by $f$, $\frac{\mu}{f} = \frac{\mu - 1}{r}$, whence

$$f = \frac{\mu - 1}{\mu} r.$$

When, on the contrary, parallel rays are incident upon the *rarer* medium, $\frac{1}{v} = 0$, and the corresponding value of $u$ being denoted by $f$, $\frac{1}{f} = -\frac{\mu - 1}{r}$, or

$$f = -(\mu - 1) r.$$

It appears therefore that a spherical surface, bounding two media of different densities, has two principal foci,—one of rays proceeding from the rarer into the denser medium, and the other of rays proceeding in the opposite direction; and that the distances of these foci from the surface are in the ratio of unity to the refractive index.

66. Defs. A *spherical lens* is a solid bounded by a plane and a spherical surface, or by two spherical surfaces. The *axis* of the lens is a right line passing through the centres of both surfaces, if both be spherical; and, when one is plane, it is the line drawn through the centre of the spherical surface, and perpendicular to the plane.

There are six forms of the spherical lens:—1. the *double-convex*, in which both surfaces are convex externally; 2. the *double-concave*, in which both are concave; 3. the *meniscus*, in which one surface is convex and the other concave, the curvature of the former being the greater; 4. the *concavo-convex*, in which one surface is convex and the other concave, the curvature of the latter being the greater; 5. the *plano-convex*; 6. the *plano-concave*.

These varieties are indicated algebraically by the signs of the radii; the radii being *positive* which are measured from the surface *towards* the incident light, and *vice versâ*.

A pencil of rays is said to be incident upon a lens *centrally*, when the axis of the pencil is coincident with the axis of the lens, and, therefore, meets both surfaces perpendicularly.

67. A small pencil of rays being incident centrally upon a lens, it is required to find the focus of the refracted pencil.

The lens being supposed denser than the medium by which it is surrounded, let $u$ and $v$ denote the distances of the radiant and conjugate upon refraction by the first surface; $v'$ and $u'$ the corresponding distances for the second; and $r$ and $r'$ the radii of the surfaces. Then (63)

$$\frac{\mu}{v} = \frac{1}{u} + \frac{\mu - 1}{r}, \quad \frac{\mu}{v'} = \frac{1}{u'} + \frac{\mu - 1}{r'}.$$

But since the focus of the pencil, after refraction by the first surface, is that of the pencil incident upon the second, we have $v = v'$, neglecting the thickness of the lens. Wherefore, subtracting the former of these equations from the latter,

$$\frac{1}{u'} - \frac{1}{u} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

This formula comprehends every case, if we observe that the positive values of $u$ and $u'$ belong to the case of *divergent* rays; and those of $r$ and $r'$ to that in which the surfaces are *concave* towards the incident light.

68. When the incident rays are *parallel*, $\frac{1}{u} = 0$, and the resulting value of $u'$ being denoted by $f$, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

The quantity $f$ is denominated the *principal focal length* of the lens. It is evident that its value is the same whichever side be turned towards the incident light. For if the lens be turned round, $r$ becomes $r'$, and *vice versâ*, and the signs of both are changed; so that the value of $f$ remains the same, in sign and magnitude, as before.

From what has been said respecting the signs of $r$ and $r'$, it is evident that the preceding formula belongs, in its actual shape, to the case of the *concavo-convex* lens, in which $r$ and $r'$ are both positive. In the *double-concave* lens, $r$ is positive and $r'$ negative, and the formula becomes

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{r} + \frac{1}{r'}\right).$$

In the *plano-concave* lens, $\frac{1}{r'} = 0$, and $\frac{1}{f} = \frac{\mu - 1}{r}$, or

$$f = \frac{r}{\mu - 1}.$$

The formulæ for the *meniscus*, the *double-convex*, and the *plano-convex* lenses agree with the preceding in everything, the sign of $f$ excepted. Hence it follows, that while the three lenses of the *concave* class cause parallel rays to *diverge* after refraction, those of the *convex* class cause them, on the contrary, to *converge*.

When the lens is of glass, $\mu = \frac{3}{2}$, and $\mu - 1 = \frac{1}{2}$, nearly. Hence, the focal length of a double-convex, or double-concave, lens of glass is an harmonic mean between its radii; and, when the radii are equal, the focal length is equal to radius. The focal length of a plano-convex, or plano-concave, lens of glass is equal to double the radius.

69. If we substitute $\frac{1}{f}$ for the second member of the general equation (67), it becomes

$$\frac{1}{u'} = \frac{1}{u} + \frac{1}{f}.$$

Now, the reciprocal of the focal distance is the measure of the divergence or convergence of the pencil. Hence, when $f$ is positive,—i. e., in lenses of the *concave* kind,—the divergence of the incident pencil is increased in the refracted, and the convergence diminished. And, on the other hand, when $f$ is negative, or the lens *convex*, the convergence is increased, and the divergence diminished.

It follows likewise from the formula, that when $f$ is negative, or the lens *convex*, the distance of the focus of the refracted pencil is *positive*, *negative*, or *infinite*, according as $u$, the distance of the focus of the incident pencil, is greater, less, or equal to $f$. Hence the refracted rays *converge*, or *diverge*, according as the incident rays diverge from a point *beyond*, or *within*, the principal focus; and when the incident rays diverge from the principal focus itself, the refracted rays are *parallel*.

70. From the formula of the preceding article we obtain

$$u' = \frac{uf}{u+f};$$

from which we learn that "the distance of the radiant from the lens is to the distance of its conjugate from the same, as the distance of the former from the principal focus of rays proceeding in an opposite direction, is to the principal focal length."

Again, subtracting $u'$ from $u$, we find

$$u - u' = \frac{u^2}{u+f};$$

i. e., "the distance of the radiant from the lens is a mean proportional between the distance of the radiant from its conjugate, and from the principal focus of rays proceeding in an opposite direction."

From the last formula we learn, that the least distance between the radiant and its conjugate is four times the principal focal length. For the quantity $\frac{u^2}{u+f}$ is least, when $u = -2f$; and its least value is therefore equal to $-4f$.

This property furnishes a simple and ready mode of determining the focal length of a convex lens experimentally. We have only to receive the image of a candle, or other bright object, upon a screen, and to move the lens and screen simultaneously, until the distance of the latter from the object is least. The one-fourth of that distance will be the principal focal length.

71. It is required to determine the focus of a central pencil after refraction by any combination of lenses in contact, whose axes are coincident.

Let $u_1$, $u_2$, $u_3$, &c., denote the distances of the foci of the refracted pencils, after refraction by the first, second, third, &c. lens; and $f_1, f_2, f_3$, &c., the principal focal lengths of the lenses. Then, since the lenses are in contact, the focal distance of the pencil, after refraction by any lens of the system, becomes that of the pencil incident upon the next; and we have

$$\frac{1}{u_1} - \frac{1}{u} = \frac{1}{f_1}, \quad \frac{1}{u_2} - \frac{1}{u_1} = \frac{1}{f_2}, \quad \frac{1}{u_3} - \frac{1}{u_2} = \frac{1}{f_3}, \text{ \&c.};$$

and adding,

$$\frac{1}{u_n} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \text{\&c.}$$

Hence it follows that the refraction produced by such a system is the same as that of a single lens, whose principal focal length is given by the formula,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \text{\&c.}$$

72. When a ray of light traverses a lens *obliquely*, the incident and emergent portions of the ray will be parallel, when the tangents to the surfaces at the points of incidence and emergence are parallel; for, in this case, the ray is under the same circumstances as if it had traversed a medium bounded by parallel planes.

Let QRR′Q′ (fig. 24) be the course of such a ray; and, C and C′ being the centres of the two surfaces, let CR, C′R′ be the radii drawn to the points of incidence and emergence. These lines, being perpendicular to the tangents at the points R and R′, are parallel. Hence, the line RR′ being produced to meet the axis of the lens in O, the triangles COR, C′OR′, are similar, and consequently,

$$\frac{C'O}{CO} = \frac{C'R'}{CR} = \frac{C'S'}{CS}.$$

Hence,

$$\frac{OS'}{OS} = \frac{C'S'}{CS}; \text{ and } \frac{OS}{SS'} = \frac{CS}{C'S' - CS}.$$

Accordingly, CS, C′S′, and SS′ being all given quantities, OS is likewise given, and the point O invariable.

All the rays which suffer no deviation pass, therefore, through a common point. This point is called the *centre of the lens;* and we learn from the preceding that its distance from the first surfarce is to the thickness of the lens, as the radius of that surface to the difference of the radii of the two surfaces.

When the thickness of the lens is inconsiderable, OS becomes indefinitely small, and the centre of the lens coincides, *q. p.*, with the surface; and, if the obliquity of the ray be small, the incident and emergent portions may be considered as parts of the same right line.

73. A pencil of rays being incident upon a thin lens, diverging from a point *near* the axis, it is required to determine the focus of the refracted pencil.

The line drawn from the focus of the incident pencil through the centre of the lens undergoes no refraction. This ray is called the *principal ray* of the oblique pencil, and is to be considered as its axis. And, the obliquity of the pencil being small, the focus of the refracted pencil will be upon this axis, and at a distance calculated by the formula,

$$\frac{1}{u'} = \frac{1}{u} + \frac{1}{f}.$$

74. Hence, when a small or distant object, PR (fig. 25), is

presented to a lens, we have only to draw lines from its several points to the centre, and on these lines, produced if necessary, to calculate the corresponding foci of refracted rays by the formula just given. The aggregate of these foci, $pr$, constitutes the *image* of the object.

When the section of the object is a *circular arc*, whose centre is the centre of the lens, $u$ is constant; $u'$, therefore, is also constant, and the section of the image will be also a circular arc having the same centre, and subtending the same angle there.

A small circular arc coincides, very nearly, with the right line which touches it at its middle point. Hence, when the section of a small object is a right line perpendicular to the axis of the lens, that of the image will be, nearly, a similar line, subtending the same angle at the centre.

75. Required the relative linear magnitudes of the object and image.

Since the angles at O (fig. 25) are equal,

$$\frac{pr}{\mathrm{PR}} = \frac{\mathrm{O}q}{\mathrm{OQ}}.$$

Wherefore, if the linear magnitudes of the object and image be denoted by $a$ and $a'$, and their distances from the lens by $u$ and $u'$,

$$\frac{a'}{a} = \frac{u'}{u} = \frac{f}{u+f}.$$

When the lens is of the *concave* kind, or $f$ *positive*, $\frac{a'}{a}$ is always less than unity, and therefore the image always less than the object, the ratio decreasing as the object recedes from the lens.

When the lens is *convex*, or $f$ *negative*, the image will be less than the object, when the distance of the latter from the lens is greater than double its focal length. When the distance is equal to double the focal length, the image is equal to the object; and when the distance is less, the image is greater, and the ratio increases indefinitely as the object approaches the principal focus. When the object is within the principal focus, the image is still greater, the ratio decreasing as the object approaches the lens; until, when the object reaches the lens, the image is again equal to it.

76. As the axes of the several pencils cross at the centre of the lens, the image will be *erect* with respect to the object, when they lie at the same side of the lens; *inverted* when at opposite sides. Hence the image of an object presented to a concave lens is always erect; while that which is formed by a convex lens is erect only when the object is between the principal focus and the lens. It is obvious that the inverted image is in all cases a *real* image, the rays actually meeting there; while the erect image is *virtual*, or imaginary.

77. The *quantity of light*, and *brightness*, of the image formed by a lens are determined by the same rules as those of an image formed by reflexion at a spherical surface; the coefficient, $p$, in this case designating the ratio of the *transmitted* to that of the incident light. Accordingly, the quantity of light in the image is as the square of the linear aperture of the lens directly, and inversely as the square of the distance of the *object*. And the brightness, or intensity of light, in the image is as the square of the linear aperture directly, and inversely as the square of the distance of the *image*. Hence, the intensity of the light in the image of the sun, formed by a convex lens, is as the square of the aperture directly, and inversely as the square of the focal length.

78. The preceding theory is exemplified in many interesting and useful applications.

If the light be admitted into a darkened chamber through an aperture in the window-shutter, and in this aperture a convex lens be placed, it is evident that inverted images of external objects will be formed within the room; and that, if the objects be at considerable distances from the lens, compared with its focal length, the distances of their images will be very nearly the same, and equal to the focal length of the lens. Hence if a sheet of paper be placed perpendicularly to the axis of the lens, at a distance equal to its focal length, an inverted picture of the external scene will be formed there, whose brightness will vary as the square of the aperture of the lens directly, and inversely as the square of its focal length.

For the purposes of drawing, it is convenient that the image should be thrown into a horizontal position. This is effected by placing a plane mirror between the lens and its principal focus, and inclined to the axis of the lens at an angle of 45°; a second image will thus be formed before the mirror, at the same distance as that which the rays tend to form behind it, and the plane of the former image is perpendicular to that of the latter (23), and therefore horizontal.

Such is the principle of the portable *camera obscura*, a box from which all extraneous light is excluded being substituted for the darkened chamber. The rays being reflected upwards, the image is received upon a piece of plane glass, roughened on one side, from which the rays will diverge as from a real object; and the extraneous light is excluded from the picture by means of a lid, with a curtain attached, which covers the head of the spectator.

In the construction described, the lower part of the mirror forms an angle of 45° with the axis of the lens: if the *upper* part be inclined to the axis at the same angle, the reflected image will be thrown *downwards*, and may be received upon a table placed at the proper distance. In this arrangement the spectator has his back turned towards the object, and views the image through an opening in the front of the box; or, as is most usual, is himself inclosed within the chamber by means of a curtain covering his person, as well as the table on which the image is thrown. This is the construction of the larger and more perfect instruments.

79. In the *magic lantern* the object is placed near the focus of the lens, and thus a distant and magnified image of it is formed upon a screen placed to receive it. The object, which is painted in transparent colours upon a plate of glass, is illuminated by a lamp, the light of which is condensed upon it by a lens of short focus; and the whole is inclosed in a box, or lantern, which intercepts all the light excepting that which goes to form the image.

The magnifying power of this instrument (75) is equal to $\frac{f}{u-f}$; and therefore, in the same instrument, varies inversely as the distance of the object from the principal focus of the lens. The brightness of the image varies as the square of the aperture of the lens directly, and inversely as the square of the distance of the image from it.

80. The principle of the *solar microscope* is the same as that of the magic lantern, the object whose image is to be represented being in this case illuminated by a beam of the sun's light, which is thrown upon it by the aid of reflectors.

It is composed of a conical tube, the smaller extremity of which is cylindrical, and furnished with a sliding tube which fits it exactly. In the extremity of the sliding tube is a convex lens of small aperture, by which the image is to be formed; and at the wider extremity of the conical tube is a broad convex lens, the use of which is to concentrate the rays of the sun upon the object, and thus to illuminate it strongly. This extremity of the instrument is inserted in a square frame, by which it may be attached to a window-shutter at an aperture corresponding to the size of the lens. To this frame is attached also, at the other side, a plane mirror, by which the light of the sun is thrown into the instrument. This mirror is moveable round a hinge, so that its inclination to the axis of the instrument may be altered at pleasure; and the hinge itself has a rotatory movement round the axis, which enables the observer to vary the plane of reflexion.

The magnifying power in this instrument is the same as in the magic lantern, being equal to the focal length of the object-lens divided by the distance of the object from its principal focus.

# CHAPTER VI.

## OF COMPOUND OR HETEROGENEOUS LIGHT.

### I. *Of the unequal refrangibility of light.*

81. WE have hitherto treated of *simple* or *homogeneous* light, and considered the modifications which it undergoes in reflexion and refraction. The light of the sun, however, and most of the lights, natural or artificial, with which we are acquainted, are *compound*, each ray consisting of an infinite number of rays differing in *colour* and *refrangibility*. This important discovery we owe to Newton. We shall briefly describe a few of the principal experiments by which it is established.

82. If a beam of solar light be admitted into a darkened room through a small circular aperture, and received on a screen at a distance, a circular image of the sun will be there depicted, whose diameter will correspond to that of the hole. If now the light be intercepted by a prism placed near the hole, having its axis perpendicular to the incident beam, the image of the sun will be thrown upwards by the refraction of the prism, and will be no longer white and circular, but coloured and oblong; the sides which are perpendicular to the axis of the prism being rectilinear and parallel, and the extremities semi-circular. The breadth of this image, or *spectrum*, as it is usually called, is equal to the diameter of the unrefracted image of the sun, but its length is much greater.

Now if the solar beam consisted of rays having all the same refrangibility, the refracted image should be circular, and of the same dimensions as the unrefracted image, from which it would differ only in position. For the rays composing the beam, being parallel at their incidence on the prism, would, on this supposition, be equally refracted by it, and therefore continue parallel after refraction.* This not being the case, we conclude that the rays composing the incident beam are of different degrees of refrangibility, the more refrangible rays going to form the upper part of the spectrum, and the less refrangible the lower; and that the elongation of the solar image, and the variety of its colouring, arises from the separation of these rays in their refraction through the prism.

From the foregoing experiment it further appears that those rays, which differ in refrangibility, likewise differ in colour; the spectrum being red at its lowest or least refracted extremity, violet at its most refracted extremity, and yellow, green, and blue, in the intermediate spaces, these colours passing into one another by imperceptible gradations. Sir Isaac Newton distinguished the spectrum, or coloured image of the sun, into seven principal colours, and determined the spaces occupied by each. These colours, arranged in the order of their refrangibility, are *red*, *orange*, *yellow*, *green*, *blue*, *indigo*, *violet;* of which the yellow and orange are the most luminous, the red and green next in order, and the indigo and violet weakest.

Any one of these rays may be separated from the rest by transmitting it through a small aperture in a screen which intercepts the remainder of the light. The ray thus separated may be examined apart from the rest, and will be found to undergo no dilatation, or change of colour, by any subsequent refractions or reflexions. We are, therefore, warranted in concluding that the solar light is *compound*, and consists of an infinite number of simple rays, which are permanent in their own nature, but differ from one another both in their *colour* and *refrangibility*.

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* In this account of the experiment it is assumed that the rays composing the solar beam are parallel. This, however, is not strictly the case, the rays proceeding from the upper and lower limbs of the sun being inclined to one another at an angle equal to the sun's apparent diameter, *i. e.*, about half a degree. It will be easily seen, however, that this difference in the incidence is wholly insufficient to account for the phenomenon: for, if the prism be placed in its position of minimum deviation, with respect to the incident beam, a small difference of incidence will produce no appreciable difference in the deviations of the emergent rays. Consequently the rays will emerge from the prism inclined at the same angle as at their incidence, if there be no difference of refrangibility; and therefore the lengths of the refracted and unrefracted images will be the same, if received upon a screen at the same distance from the hole.

83. To the preceding conclusion, however, it was objected that *each* ray of the incident beam is, by the action of the glass, *dilated* or *split* into many diverging rays, without any difference in refrangibility; or, supposing a difference of refraction to exist, still that inequality may be *fortuitous*, and not the result of any inherent diversity in the rays themselves.

To overthrow these objections the following experiments are adduced by Newton:

I. Let the light which emerges from the prism be received on a second prism, whose axis is *parallel* to that of the first, and whose refracting angle is equal and turned in the opposite direction. Then, if the length of the solar image arose from a splitting or dilatation of the ray by the first prism, the second prism should produce an equal effect, and the total length of the image should be *doubled*. In place of this, the image produced by the two prisms is found to be *round* and *colourless*, all the rays being refracted equally, and in opposite directions, by the two prisms.

II. Again, let the light emerging from the first prism be received on a second, whose axis is *perpendicular* to that of the first, and therefore the planes of their refractions at right angles. If, now, the length of the solar image arose from a dilatation of each ray, or from any casual inequality in their refractions, it is evident that the oblong image produced by the first refraction should, by a second refraction made laterally, be increased in breadth, as it was before in length by the same causes, and should, therefore, appear square or rectangular. But the result is found to be otherwise. The image is not at all increased in breadth by the refraction of the second prism, but only becomes *oblique* to its former position, the upper or violet extremity being translated farther from its former position than the lower or red extremity. Thus the light which is most refracted by the first prism is again most refracted by the second; and that which is least refracted by the first is, in like manner, least refracted by the second. And since the sides of the oblique image are found to be rectilinear, as well as those of the first, it follows that every ray which is more or less refracted by the first prism, is, exactly in the same proportion, more or less refracted by the second.

III. The following experiment, however, may be considered as removing all doubt on this subject. Close behind the prism is placed a board, perforated with a small aperture, through which the refracted light is permitted to pass. This light is then received on a second board, at a considerable distance from the first, and similarly perforated; so that a small portion of the light of the spectrum is suffered to pass through the aperture in the second board, the rest being intercepted. Close behind this aperture a second prism is fixed, having its axis parallel to that of the first, by which the transmitted light is a second time refracted. The first prism being then turned slowly round its axis, the light of the spectrum will move up and down on the second board, and the differently-coloured rays be successively transmitted through the aperture in it, and fall upon the prism behind it. If then the places of the differently-coloured rays on the screen be noted, the red will be found to be lowest, the violet highest, and the intermediate colours in the same order as they are in the spectrum. Here, on account of the unchanged position of the apertures, all rays are necessarily incident upon the second prism at the same angle; and yet some of them are more refracted and others less, in the same proportion as by the first prism.

From the foregoing we conclude, then, that the peculiar colour and refrangibility belonging to each kind of homogeneous light, are permanent and original affections, and are not generated by the changes which that light undergoes in refractions or reflexions.

84. In the experiments hitherto described, the analysis of solar light, or its resolution into its simple components, is far from being complete, inasmuch as there is a considerable mixture of the different species of simple light in the coloured image of the sun produced by the prism. This will be evident, if we consider that, as the several pencils of *homogeneous* light suffer no dilatation by the prism, each will depict on the screen a circular image equal in magnitude to the unrefracted image of the sun at the same distance. Hence the spectrum consists of innumera-

ble circles of homogeneous light, whose centres are disposed along the same right line, and whose common diameter is that of the sun's unrefracted image. Wherefore the number of such circles mixed together in the spectrum is to the number mixed together in the unrefracted image of the sun, as the interval between the centres of two contingent circles (or the breadth of the spectrum), to the interval between the centres of the extreme circles, which is the length of the rectilinear sides. The mixture in the spectrum, therefore, varies as the breadth of the spectrum divided by its length; and if the breadth can be diminished, the length remaining the same, the mixture will be diminished in proportion.

There are various ways of diminishing the breadth of the spectrum, or the diameter of the sun's unrefracted image, amongst which that of Newton seems as convenient in practice as any. The beam of solar light, admitted into a darkened chamber through a small circular aperture, is received upon a lens of considerable focal length, at the distance of double its focal length from the aperture; at the same distance beyond the lens will be formed a distinct image of the hole, equal to it in magnitude. A prism being then placed immediately behind the lens, this image will be dilated in length, its breadth remaining unaltered, and thus a spectrum will be formed whose breadth is the diameter of the hole; whereas, without this contrivance, the breadth would be equal to that diameter together with a line which, at the distance of the screen from the hole, subtends an angle equal to the apparent diameter of the sun. Thus, by diminishing the diameter of the hole, the breadth of the spectrum, and therefore the mixture, may be reduced at pleasure.

If the diameter of the aperture be very small, the spectrum is reduced to a narrow line, and is unfit for examination. To remedy this inconvenience, Newton employed a narrow rectangular aperture, whose length is parallel to the axis of the prism, and may be as great as we please, while its breadth is very small. In this manner we obtain a spectrum as broad as we wish, and whose light is, notwithstanding, as simple as before.

85. It is required to determine, experimentally, the index of

refraction of the several species of simple light of which solar light is composed.

The method employed by Newton consisted in determining the refractive indices of the extreme red and violet rays directly by means of the formula of (60), from which those of the other rays may be obtained by a simple proportion.

Thus, if $\delta_1$ and $\delta_2$ denote the minimum deviations of the extreme red and violet rays, and $\mu_1$ and $\mu_2$ their refractive indices, we have

$$\mu_1 = \frac{\sin \frac{1}{2} (a + \delta_1)}{\sin \frac{1}{2} a}, \quad \mu_2 = \frac{\sin \frac{1}{2} (a + \delta_2)}{\sin \frac{1}{2} a}.$$

Now the difference of the angles $\delta_1$ and $\delta_2$, which is the total dispersion, is obtained by dividing the interval between the centres of the extreme circles (or the difference between the length and breadth of the spectrum), by its distance from the prism. And the sum of these angles, which is double the deviation of the mean ray, is obtained by measuring the altitude of the sun, and the inclination of the mean ray to the horizon at emergence. The sum and difference of the angles being thus obtained, the angles themselves are determined; and, their values being substituted in the preceding formulæ, the refractive indices of the extreme rays are known.

In this manner, when the refracting prism was of crown-glass, Newton found for the indices of refraction of the extreme red and violet rays,

$$\mu_1 = \tfrac{77}{50}, \quad \mu_2 = \tfrac{78}{50}.$$

To determine the refractive indices of the several intermediate rays, it was necessary to determine the spaces which they occupied respectively in the coloured image of the sun. For this purpose Newton delineated on paper the spectrum AH*ha* (fig. 26), and distinguished it by the cross lines A*a*, B*b*, C*c*, &c., drawn at the confines of the several colours; so that the space AB*ba* is that occupied by the red light, BC*cb* that by the orange, CD*dc* the yellow, DE*ed* the green, EF*fe* the blue, FG*gf* the indigo, and GH*hg* the violet. He then found that, if the whole length of the rectilinear side AH be taken as unit, the distances to the confines of the several colours AB, AC, AD, &c., will be denoted by the numbers $\frac{1}{8}$, $\frac{1}{5}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{9}$, 1; the refracting prism being of crown-glass, as before. Now the intervals AB, BC, CD, &c., occupied by the several colours in the spectrum, or the differences of the deviations which they subtend, are to one another as the corresponding variations of $\mu$, the index of refraction. If, therefore, the whole variation of $\mu$, or $\frac{1}{50}$, be divided as the line AH is in the points B, C, D, &c., the refractive indices of the rays at the confines of the several colours will be as follow:

$$\frac{77}{50}, \quad \frac{77\frac{1}{8}}{50}, \quad \frac{77\frac{1}{5}}{50}, \quad \frac{77\frac{1}{3}}{50}, \quad \frac{77\frac{1}{2}}{50}, \quad \frac{77\frac{2}{3}}{50}, \quad \frac{77\frac{7}{9}}{50}, \quad \frac{78}{50}.$$

The *mean* refractive index is $\frac{77\frac{1}{2}}{50}$, or 1·55; and it appears from the preceding that this belongs to the rays at the confines of the green and blue.

86. On a minute examination of the solar spectrum, when every care has been taken in making the experiment, it is found that it is not, as Newton supposed, a continuous band of coloured light, whose intensity is greatest about the confines of the yellow and orange, and diminishes regularly to the two extremities; but that, on the contrary, there are, at certain points, *abrupt deficiencies of light*, total or partial, indicated by the existence of numerous dark streaks, or *bands*, crossing the spectrum in the direction of its breadth; and that, in the intermediate spaces, the intensity of the light does not increase or decrease continually, but varies irregularly, or according to some very complex law. Solar light, then, does not consist (as has been hitherto supposed) of rays of every possible refrangibility within certain limits; for it is found that many rays, corresponding to certain degrees of refrangibility, are wanting in the spectrum.

Some of these bands are wholly black, others dark, of various degrees of illumination; they differ also from one another in breadth, and are irregularly disposed throughout the length of the spectrum. They are not, however, the result of any accidental cause; for, when solar light is used, and the refracting substance is the same, it is found that they preserve the same relative position, both with respect to one another and to the colours of the spectrum. On the other hand, when the refracting substance is varied, their relative positions *with respect to one another* are altered; but their positions as referred to the colours of the spectrum, as also their relative breadth and intensity, remain unchanged.

If other kinds of light, as that of the fixed stars, flames, &c., are examined in the same way, similar bands are discovered, but differing in each species of light in their position, &c.; so that each species of flame, and the light of each fixed star, has its own system of bands, which remains unalterable under all circumstances, and which, therefore, is a distinct physical characteristic of the species of light to which it belongs.

These *fixed lines*, as they are called, were first noticed by Wollaston, in the year 1802. They have since been much more fully examined by Fraunhofer, who distinguished more than 700 in the solar spectrum, of which he has selected seven principal ones, to serve as standards of comparison. Two of these are in the red, the third in the orange, and the remaining four in the green, blue, indigo, and violet, respectively. They are of the utmost importance in optical investigations, inasmuch as they preserve an invariable position with respect to the colours of the spectrum, and the accuracy of their delineation renders their observation susceptible of the utmost nicety. In the method of determining the refractive index of the different species of homogeneous light, already described, it is impossible to fix with precision the position of any particular ray in the coloured image, and therefore to determine, with accuracy, its index of refraction. The position of the fixed lines, however, may be observed with an accuracy equal to that of astronomical measurements, and thus the refractive indices of the rays, to which they correspond, determined with the utmost exactness.

87. As the white light of the sun is, by the foregoing experiments, resolved into its simple elements, so these latter may be again compounded, and thus made to reproduce a white light agreeing in all its properties with the light directly received from the sun.

To effect this, the light refracted by one prism may be received on a second at a small distance from it, the refracting angle of the second prism being equal to that of the first, and turned in an opposite direction. Under these circumstances, all the rays—being equally deviated in opposite directions by the two prisms—will emerge in a direction parallel to that of the rays incident upon the first, and will therefore be mixed together. It is accordingly found that the light, which is coloured between the two prisms, emerges colourless from the second.

Again, if the spectrum be received upon a broad lens at the distance of double its focal length from the prism, the coloured rays diverging from the prism are made to converge to a focus, at the same distance beyond the lens, and there by their mixture produce a perfect whiteness,—as will be evident by receiving them on a sheet of white paper at that distance. If the paper be removed to a greater distance from the lens, the colours reappear in the contrary order; showing plainly that the whiteness at the focus is the result merely of the *mixture* of the variously-coloured rays, which converge to that point, and, crossing there, again diverge. The divergence of this beam may be corrected by means of another prism, whose refracting angle is equal to that of the former, placed at the focus where the rays are mixed. Thus the coloured rays are reduced to parallelism, and the emergent beam compounded of them is perfectly white. This beam may be subjected to experiment, just as the direct light of the sun, and is found to be possessed of all the same properties; and if by a new refraction it be again resolved, and any of its component elements then stopped at the lens, the same colour will disappear from the refracted image. It is, therefore, abundantly evident, that these several refractions produce no other effect than that of mixing or separating the rays of simple light, without impressing any new modification upon them, or changing in any respect their colorific qualities.

88. The coloured lights reflected from natural bodies may be in like manner compounded, and, if mixed together in the same proportion in which they enter solar light, will compose a white-

ness which will be more or less perfect in proportion to the vividness of the colours employed.

Thus if, by lines drawn from the centre, the area of a circle be divided in the proportion of the coloured spaces of the spectrum, and these sectors be painted with the prismatic colours, it is found that, by a rapid rotation of this circle round an axis passing through the centre, and perpendicular to its plane, the eye will be affected with the sensation of whiteness. Now, as the effect of any impression on the retina continues for some short time, a rapid rotation will produce the same effect, as to vision, as if the several colours had been mixed together in the same object, and thus impressed simultaneously; and this effect, it is seen, is the sensation of whiteness.

### II. *Of the unequal reflexion of light, and of the colours of natural bodies.*

89. We have hitherto spoken of the reflexions and refractions which light undergoes when it encounters an even surface, such as the surfaces of liquids when at rest, and the artificial surfaces given to glasses or metals by grinding and polishing; and have seen that the rays of light are reflected or refracted regularly in certain directions, so as to meet the eye only when placed in those directions. Such, however, is not the case with the inartificial surfaces of most natural bodies. These, on account of their inequalities, present every inclination of surface to the incident light; and any portion of them, however small, may be regarded as a polyhædron of an indefinite number of sides. Accordingly, the light incident upon every such portion will be reflected in every possible direction. The reflected light, therefore, radiates from every part of such a body,—as the direct light from a self-luminous body,—in all directions; and is visible to the eye wherever placed.

90. There is no body in nature which reflects the whole of the light incident on it; a part of the light, in all cases, enters the body, and is either transmitted or absorbed. The *brightness*

of natural bodies depends upon the proportion which the light reflected by them bears to the incident light.

But further, bodies not only differ from one another in the whole quantities of solar light which they reflect; but also, in one and the same body, the proportion of the reflected to the incident light is, in general, different for each species of simple light of which solar light is composed. Hence it is that natural bodies appear *coloured*, the colours which they exhibit being those of the predominant rays in the light reflected by them; and, accordingly, the colours of natural bodies arise from their aptitude to reflect this or that species of homogeneous light more copiously than others.

That such is a just explanation of the phenomena of the colours of natural bodies appears fully from the experiments of Newton. All bodies, whatever be their colour when exposed to solar light, exhibit the colour of the homogeneous light in which they are placed; appearing brighter, however, and more luminous in the light of their own colour, than in any other. Thus, *cinnabar* appears red when placed in a homogeneous red light, green in a green light, and blue in a blue; its brightness, however, being greatest in the red light, less in the green, and least of all in the blue. *Ultramarine*, on the contrary, appears brightest when placed in a homogeneous blue light; that brightness diminishes when it is brought into the green light, and is least of all in the red ; the colour which it exhibits being, however, in every case, that of the light in which it is placed. Again, if cinnabar and ultramarine be compared together in different lights, it will be found, that when placed in a homogeneous red light, they both appear red,—the cinnabar being of a brilliant red, the ultramarine of a very obscure one; but when transferred to the blue extremity of the spectrum, the order of their brightness is reversed, the cinnabar reflecting an obscure blue, and the ultramarine a brilliant one. From all which it is evident that the cinnabar reflects the red rays more copiously than those of any other colour, and the ultramarine the blue; and so of other bodies.

91. Bodies which are of a *white*, *grey*, or any *neutral* colour, such

as those which the clouds exhibit, reflect *all* the different species of light in the *same proportion*, the intensity of the shade depending upon the reflecting power. All such shades differ from one another in degree only, not in kind; the darkest grey differing from the most brilliant white only in the quantity of light. This was fully evinced by Newton, by comparing such bodies together under different degrees of illumination.

White bodies reflect the light of any particular colour more copiously even than bodies of that colour, all coloured bodies absorbing a considerable portion of the incident light.

Bodies which exhibit a *homogeneous* light of any colour, reflect only the light of that particular colour. Such bodies, however, are not to be met with in nature; amongst those which approach nearest to them may be reckoned blood, gamboge, and ultramarine. Bodies of the most vivid prismatic colours, such as minium and vermilion, generally reflect a considerable portion of each of the different kinds of light, with a predominant portion of the light of their own colour.

# CHAPTER VII.

## OF THE DISPERSION OF LIGHT.

92. It has been shown, in the preceding chapter, that the refractive index of a given substance is different for each of the species of simple light of which solar light is composed, its magnitude varying within certain limits dependent on the nature of the refracting substance; and that the least of these limiting values belongs to the extreme red ray of the spectrum, and the greatest to the extreme violet. Accordingly, the fundamental law of refracted light, namely,

$$\sin\phi = \mu \sin\psi,$$

may be extended to heterogeneous or compound light, by considering the refractive index $\mu$ as a variable quantity, whose different values belong to the different species of simple light of which the ray is composed.

93. The dispersion of a ray of light, produced by refraction at a single surface, varies as the tangent of the angle of refraction.

Let $\psi_1$ and $\psi_2$ denote the angles of refraction of the extreme red and violet rays, $\mu_1$ and $\mu_2$ the refractive indices,

$$\sin\psi_1 = \frac{\sin\phi}{\mu_1}, \quad \sin\psi_2 = \frac{\sin\phi}{\mu_2}, \text{ and } \sin\psi_2 - \sin\psi_1 = \left(\frac{1}{\mu_2} - \frac{1}{\mu_1}\right)\sin\phi.$$

But $\sin\psi_2 - \sin\psi_1 = 2\sin\frac{1}{2}(\psi_2 - \psi_1)\cos\frac{1}{2}(\psi_2 + \psi_1) = (\psi_2 - \psi_1)\cos\psi_1$, *q. p.*, since $\psi_2 - \psi_1$ is small. Hence

$$\psi_2 - \psi_1 = \frac{\mu_1 - \mu_2}{\mu_2}\tan\psi_1.$$

When the course of the ray is from the denser into the rarer medium, it will appear by a similar reasoning that the deviation is

$$\phi_2 - \phi_1 = \frac{\mu_2 - \mu_1}{\mu_1} \tan \phi_1.$$

94. Required the dispersion of a ray which passes nearly perpendicularly through a thin prism.

If $\delta_1$ and $\delta_2$ denote the deviations of the red and violet rays, $\mu_1$ and $\mu_2$ the refractive indices of the prism for those rays, and $a$ its refracting angle, we have

$$\delta_1 = (\mu_1 - 1)\, a, \quad \delta_2 = (\mu_2 - 1)\, a;$$

and

$$\delta_2 - \delta_1 = (\mu_2 - \mu_1)\, a.$$

Accordingly, the dispersion, in this case, is equal to the angle of the prism multiplied by the difference of the refractive indices.

95. Required the measure of the dispersive power of any medium.

The dispersive power of a substance is measured,—not by the absolute dispersion, which varies in general with the angle of refraction,—but by the ratio which that quantity bears to the total deviation, or by $\frac{\delta_2 - \delta_1}{\delta_1}$. But, in the case of a ray which passes nearly perpendicularly through a thin prism, this ratio is constant; for, dividing the third of the equations of the preceding article by the first,

$$\frac{\delta_2 - \delta_1}{\delta_1} = \frac{\mu_2 - \mu_1}{\mu_1 - 1}.$$

The dispersive power, therefore, is measured by the difference of the refractive indices of the red and violet rays, divided by the refractive index of the former *minus* unity.

96. Newton supposed that the dispersive powers of all substances were the same,—or, in other words, that the dispersion was in all cases proportional to the total deviation. He was led to this erroneous conclusion by observing, that when a prism of

glass was inclosed in a prism of water with a variable angle, their refracting angles being turned in opposite directions, the emergent ray was *coloured* when it was *inclined* to its original direction; while, on the other hand, it was *colourless* whenever, by varying the angle of the water prism, the refractions of the two prisms were made to compensate each other, or the ray to emerge parallel to the incident ray. Hence he concluded that the dispersion was always proportional to the total deviation; and therefore that refraction could never take place without a separation of the refracted ray into its coloured elements. Now this dispersion being the chief cause of the imperfection of refracting telescopes, Newton was led, by the result of this experiment, to despair of the improvement of these instruments, and to turn his attention to those in which the image was formed by reflexion instead of refraction.

When Newton's experiment with the two prisms was repeated, a long time after, by Dollond, he was surprised to find that the results were exactly the opposite to those stated by Newton;—that, in fact, the emergent ray was *coloured*, when the deviation was nothing, or the ray parallel to its original direction;—and that, on the other hand, when the dispersions of the two prisms were made to correct each other by varying the angle, or the ray to emerge *colourless*, their refractions were no longer equal, and the ray emerged *inclined* to its original direction. This important discovery led to the construction of the achromatic telescope.

97. A ray of light passes nearly perpendicularly through two prisms, whose refracting angles are small; required the condition of achromatism.

The dispersions produced by the two prisms are $(\mu_2-\mu_1)\,a$, and $(\mu_2'-\mu_1')\,a'$, respectively (94); and, therefore, when the total dispersion is nothing, we must have

$$(\mu_2-\mu_1)\,a+(\mu_2'-\mu_1')\,a'=0, \quad \text{or } \frac{a'}{a}=-\frac{\mu_2-\mu_1}{\mu_2'-\mu_1'}.$$

The negative sign, in the second member, indicates that the angles of the two prisms must be turned in opposite ways.

98. It is required to compare experimentally the dispersive powers of different substances.

In order to ascertain the relative dispersive powers of different substances, they must be separately compared with that of some standard substance, such, *e. g.*, as water. For this purpose a vessel must be constructed, whose opposite sides, formed of parallel glass, are moveable on hinges, and may be inclined to one another at any angle. It is closed on the other two sides by metallic cheeks, to which the moveable sides are accurately fitted. The vessel being filled with water, it is evident that the transmitted ray will be refracted in the same manner as by the inclosed water prism, the parallel plates of glass producing no change in the direction of the refracted ray (51). The substance whose dispersive power is sought being formed into a thin prism, a beam of light is to be transmitted nearly perpendicularly through the two prisms, with their refracting angles turned in opposite directions; and the angle of the water prism is to be varied, until the beam emerges colourless. The angle of the water prism being then measured, the ratio of the differences of the refractive indices (and thence that of the dispersive powers), will be given by the formula of the preceding article.

A solid prism of a variable angle may be constructed by joining together two plano-cylindrical lenses, of equal curvatures, one of which is plano-convex, and the other plano-concave. The convex surface fitting in the concave, and revolving round the common axis of the two cylinders, it is obvious that the plane surfaces may be inclined at any angle.

99. The dispersion produced by a lens varies as the aperture directly, and inversely as the principal focal length.

The refraction produced by a lens, at any point, is the same as that of a prism whose refracting angle is equal to the angle contained by the tangent planes at the points of incidence and emergence. Let POR (fig. 27) be the axis of the lens, P the focus of the incident pencil, and PA the extreme ray. The incident light being compound, the refracted light corresponding to the ray PA will be *dispersed*, the red ray being refracted in the direction AR, the violet ray in the direction AV, and the intermediate rays of the spectrum in intermediate directions. The angle QAR, therefore, is the deviation of the red ray, QAV that of the violet ray, and their difference, RAV, is the angle of dispersion. And (95) there is

$$\text{RAV} = \text{QAR} \times \frac{\mu_2 - \mu_1}{\mu_1 - 1}.$$

But $\text{QAR} = \text{APO} + \text{ARO}$; and these angles, being small, are measured by their common subtense, AO, divided by the distances OP and OR, respectively. Hence

$$\text{QAR} = \text{AO}\left(\frac{1}{\text{OP}} + \frac{1}{\text{OR}}\right) = \text{AO}\frac{1}{f};$$

$f$ being the principal focal length of the lens. Accordingly, if the semiaperture of the lens, AO, be denoted by $A$, and its dispersive power, $\frac{\mu_2 - \mu_1}{\mu_1 - 1}$, by $\pi$, the dispersion is equal to

$$\frac{\pi A}{f}.$$

100. It is manifest from the preceding that the foci of refracted rays will be different for each of the different species of simple light of which solar light is composed, the more refrangible rays being brought to a focus at a point nearer to the lens, and the less refrangible at a point more remote. The portion of the axis, RV, occupied by these foci, is called the *chromatic aberration;* and if the extreme violet rays, AV, A′V (fig. 28), be produced to meet the extreme red rays, AR, A′R, in $a$ and $a'$, it is obvious that $aa'$ is the diameter of the *least space* into which all the rays are collected. This space is called the *circle of chromatic aberration;* and it is to be regarded as the physical focus of the refracted pencil.

101. Required the diameter of the circle of chromatic aberration.

$aa'$ (fig. 28) is equal to $a\text{A}a'$ multiplied by AV or OV. Wherefore, denoting the focal length OV by $u'$, and substituting for the dispersion, $a\text{A}a'$, its value $\frac{\pi A}{f}$, there is

$$aa' = \pi A \cdot \frac{u'}{f}.$$

When the incident rays are parallel, $u' = f$, and there is, simply,

$$aa' = \pi A.$$

For parallel rays, therefore, the diameter of the circle of aberration is equal to the semiaperture of the lens, multiplied by the dispersive power.

102. It has been shown that two prisms, of different substances, may be combined so as to correct each other's dispersions, and nevertheless to produce a deviation in the emergent ray. In like manner, a convex and concave lens, of different substances, may be combined so that the refraction of one shall predominate, and the rays be brought to a focus, while, at the same time, the dispersions of the two lenses are equal and opposite, and the emergent pencil is colourless. Such a combination is called *achromatic*.

103. Two lenses in contact will form an achromatic combination, provided one of them be convex and the other concave, and their focal lengths proportional to their dispersive powers.

Let $f$ and $f'$ denote the focal lengths of the two lenses, $\pi$ and $\pi'$ the dispersive powers of the substances of which they are composed; then (99) the actual dispersions are $\frac{\pi A}{f}$ and $\frac{\pi' A}{f'}$, $2A$ being the common aperture. And the total dispersion, therefore, is

$$\left(\frac{\pi}{f} + \frac{\pi'}{f'}\right)A.$$

Accordingly the combination will be achromatic when the quantity within the brackets = 0, that is, when

$$\frac{\pi}{f} = -\frac{\pi'}{f'}.$$

Hence the focal lengths must be of opposite signs, and proportional to the dispersive powers.

The substances usually employed in the construction of the achromatic lens are the species of glass denominated *crown glass* and *flint glass*, whose dispersive powers are in the ratio of 2 to 3, nearly. Hence, if we put together a convex lens of crown glass and a concave of flint glass, whose focal lengths are in this proportion, the combination will be achromatic.

The compound lens will be equivalent to a single convex lens, whose focal length is triple the focal length of the crown glass lens. For, if $f$ denote the focal length of the convex lens, that of the concave will be $\frac{3}{2}f$; and the former being negative, the focal length of the compound (71) will be given by the formula

$$\frac{1}{F} = \frac{2}{3f} - \frac{1}{f} = -\frac{1}{3f}; \text{ whence } F = -3f.$$

# CHAPTER VIII.

## OF THE HUMAN EYE.

104. Figure 29 represents a horizontal section of the human eye. Its form is nearly spherical, the fore part, ABA′, however, being more convex than the rest.

The several humours of which the eye consists are all contained in a thick, tough coat, the more convex part of which, ABA′, called the *cornea*, is transparent, and of a consistent horny character. The remainder of this exterior coat is called the *sclerotica:* it is opaque and white, and forms what is in common language designated as the *white of the eye.*

The interior of the sclerotica is lined by a second and thinner coat of a softer substance, called the *choroid membrane*, which is firmly attached to the sclerotica by a circular band extending round the edge of the cornea, and called the *ciliary ligament.*

At the junction of the choroid with the sclerotica, and supposed to be a continuation of the former membrane, arises the *uvea*, an opaque membrane or screen, having an aperture in the centre, *aa*′, called the *pupil*, through which the rays of light incident on the eye are admitted.

The uvea consists of muscular fibres, by the contraction or expansion of which the aperture of the pupil is contracted or enlarged. The use of this is to moderate the quantity of light incident upon the sensitive part of the eye. In very strong lights the pupil is contracted, in weak ones it is enlarged; its aperture varying, in the eyes of adult persons, from about one-tenth to one-fourth of an inch. This contraction and dilatation of the pupil is involuntary, and arises on the excess or defect of the light itself.

The anterior surface of the uvea is differently coloured in different persons, varying through all the shades of green, blue, brown, and grey. This part is sometimes called the *iris*, and its colour determines that of the eye.

The interior of the choroid membrane is covered with a very black mucous substance, called the *pigmentum nigrum*, in which is imbedded the *retina*, the third and innermost coat of the eye. This is a network of extremely fine nerves, branching from the optic nerve, O, which proceeds directly from the brain, and enters obliquely at the back of the eye, and at the inner side towards the nose.

105. Within the eye, and a little behind the uvea, is suspended a soft, transparent, jelly-like substance, CC′, called the *crystalline lens*, of the form of a double-convex lens of unequal radii, the anterior surface being less curved than the posterior. The structure of this substance is fibrous, being composed of laminæ or layers successively superimposed, as may be seen in the lens of a boiled fish's eye; and each coat consists of a vast number of fibres diverging from two poles, the line joining which coincides with the axis of the eye. The crystalline is contained in a thin transparent capsule; and kept in its place by the *ciliary processes*, a projecting fold of the choroid membrane, which arises at the same place as the uvea, and is a little convex towards it.

The space before the crystalline humour, and between it and the cornea, is filled with a transparent fluid resembling water, and thence denominated the *aqueous humour*. The space behind the crystalline, and between it and the retina, is filled with another transparent fluid somewhat more viscous than the former, and called the *vitreous humour*. These two humours are, like the crystalline, contained in transparent membranous capsules of extreme delicacy and tenuity.

In their refractive powers the aqueous and vitreous humours differ very little from that of water. The refractive index of the aqueous humour is 1·337, and that of the vitreous humour 1·339; that of water being 1·336. The refractive power of the crystalline is greater, its mean refractive index being 1·384. The density of the crystalline, however, is not uniform; but increases gradually from the outside to the centre. This increase of density serves to correct the aberration, by increasing the convergence of the central rays more than that of the extreme parts of the pencil.

The axis of the human eye, measured from the outer surface of the cornea to the retina, is about ·95 of an inch; and the portions of it occupied by the cornea and the different humours are as follow: cornea, ·04 of an inch; aqueous humour, ·11; crystalline, ·17; vitreous humour, ·63. Hence the portion occupied by the vitreous humour is about two-thirds of the whole length of the axis.

106. Having thus far explained the structure of the eye, we proceed, in the next place, to consider it as the instrument of vision.

The eye, we have seen, consists of three refracting media, of which the two extremes have nearly the same refractive power as water, and the intermediate one a refractive power somewhat greater. Accordingly, the light incident from any object on the eye will undergo a refraction at each of the bounding surfaces of these media, and this refraction in each case tends to give convergence to the incident rays. Thus the first refraction takes place at the convex surface of the aqueous humour, which, being denser than the surrounding air, will give a convergence to the refracted pencil. The extreme rays of this pencil are then intercepted by the uvea; and the central part of the pencil, being transmitted through the pupil, is incident upon the convex surface of the crystalline, which, being denser than the aqueous humour, increases the convergence; and, finally, it falls upon the concave surface of the vitreous humour, which, being rarer than the medium from which the pencil has emerged, adds to the convergence, and completes the refraction. By means of these successive refractions each pencil of incident rays is brought to a focus at or near the retina, and thus an image is formed there corresponding in form and colour with the object fron which the rays emanate. These images may be readily exhibited by taking off the outer coats from the back of the eye of a newly killed animal: miniature pictures of external objects will be seen depicted there, as upon a screen of roughened glass.

It is evident that the axes of the several pencils intersect before they reach the retina; and therefore that the image on the retina will be inverted with respect to the object. The point in which these axes intersect is called the *focal centre* of the eye. Its position, it is obvious, will vary both with the distance of the object and its magnitude; but it is never far distant from the posterior surface of the crystalline.

107. The *apparent magnitude* of any object is measured by the angle contained by the axes of the extreme pencils; that is, by the angle subtended by the object at the focal centre. It is therefore proportional to the linear magnitude of the object divided by its distance from the centre of the eye.

There has been much discussion with respect to the magnitude of the *minimum visibile*, or the apparent magnitude of the least visible object. It is usually stated that a single object upon a ground of an opposite colour,—as a black circle upon a white ground, or a white circle upon a black ground,—cannot be seen by the generality of eyes under a smaller angle than one minute. It would appear, however, that the power of exciting the sensation of vision depends, not on the magnitude of the *visual angle*, but on the *quantity of light* proceeding from the object in relation to that proceeding from surrounding objects. Many facts may be adduced in support of this; and if it be admitted, it follows that the angle subtended by the least visible object may be indefinitely diminished, if its brightness be proportionally increased. Accordingly we find that the fixed stars produce a distinct and vivid impression on the retina, although the angle they subtend at the eye is less than any we are able to estimate by the most accurate observations.

This conclusion is perfectly compatible with the supposition that the image on the retina must have some definite magnitude in order to produce an impression. We have strong reasons for believing that the eye is not free from chromatic dispersion; and from this it follows, even when the light is collected to a focus exactly at the retina, that the image of a point will be diffused there over a small circle of a definite magnitude. And, as the density of the light in this circle diminishes from the centre to the circumference, the extent of it which is capable of impressing the retina will depend upon the whole quantity of light emanating from the luminous point. This will explain the reason why the brighter fixed stars appear larger than those which are fainter, although the angle subtended by none is of any definite magnitude.

108. The greatest extent of field which the eye is capable of receiving at once, is measured by the angle which the diameter of the pupil subtends at the focal centre of the eye, the retina being supposed sensible to the impressions of light within these limits. This extent of field, however, is found to be different in different directions. Dr. Young found that, when the visual axis was fixed in a certain direction, the angle of vision of his own eye extended upwards 50°, downwards 70°, inwards 60°, and outwards 90°. This great extent of field is due to the position of the uvea. Had it been placed in front of the eye, or at any considerable distance from its present position, it must have greatly limited the field of view, which is measured by the angle which the aperture in the uvea, or pupil, subtends at the focal centre of the eye; and if, to increase the field of view, this aperture had been enlarged, it would not have served its present purpose of intercepting the extreme rays. But, in its present position, which is not far distant from the focal centre, it not only intercepts the extreme rays of each pencil, and thus corrects the aberration, but also admits pencils whose axes contain a large angle, and therefore opens a wide field of view to the eye.

As the uvea itself suggested, in all probability, the use of the *diaphragm*, or eye-stop, in telescopes, so this peculiar propriety of its position has been advantageously imitated by Dr. Wollaston in the construction of his *periscopic* lenses. These consist of two plano-convex lenses, united together at their plane sides, between which is interposed a diaphragm, having an aperture in the centre. The field of view is obviously much greater than in lenses whose aperture is equal to that of the diaphragm.

It must not be supposed, however, that the vision is distinct throughout the whole of the wide field of view which the eye possesses. Whether it arises from the want of perfect sensibility in the parts of the retina remote from the visual axis, or from the distortion of oblique pencils, or from both these causes conjointly, it is found that those objects alone are seen *distinctly* which are in the centre of the field; the remaining parts of the field serving merely to convey notices to the mind of the *presence* of objects situated there. The field of *perfect vision* does not extend more than 5° from the visual axis in every direction.

This limitation of the field of distinct vision, however, is attended with no practical disadvantages of any moment; for, by the revolution of the eye in its orbit, the axis of the eye has a range of 55° in every direction; and by these means the field of perfect vision has an actual extent of 110°, independently of any motion of the head or of the rest of the body.

109. It is a curious fact that there exists a small portion, within the limits of the field of view of the eye, which is invisible; or, in other words, there is a part of the back of the eye which is insensible to the impression of the rays of light, and which for that reason has been called the *punctum cæcum*. It is that spot at which the optic nerve is introduced, and at which that nerve is not yet subdivided into those delicate and sensitive fibres of which the retina is composed. This may be observed by placing two patches of white paper upon a dark wall, the line joining them being horizontal, and about the height of the eye from the ground. If then one eye be closed, and the other directed to one of the patches (the one to the left hand if the right eye be used, and *vice versâ*), the other, to which the eye is not directed, becomes invisible on retiring from the wall to about four or five times the distance of the patches from one another, and, the distance being further increased, becomes again visible.

110. The *apparent brightness* of any luminous object is proportional to the density of the light in its image on the retina,

which is as the quantity of light directly, and inversely as the space over which it is diffused. Now, if the aperture of the pupil be supposed invariable, and no light be lost in its passage through the air, the quantity of light incident on the pupil, and therefore that which falls on the retina, varies inversely as the square of the distance of the object (13); and it is evident (107) that the area of the image on the retina, or the space of diffusion, varies in the same ratio. Hence, on these suppositions, the brightness of a luminous object is the same at all distances from the eye.

In actual experience, however, we find that the brightness of all luminous objects diminishes with the distance, and that at considerable distances they become altogether invisible. The reason of this is, that the air is not a perfectly transparent medium, but on the contrary absorbs a considerable portion of the transmitted light.

111. If the form of the eye were *invariable*, it is evident that it would collect such rays only to a focus on the retina which had a *certain degree* of divergence at their incidence on the eye;—that rays incident with a less divergence, or proceeding from remoter objects, would be brought to a focus before they reached the retina, while those which proceeded from a nearer distance would tend to a focus beyond it;—and that in either case the image on the retina itself would be confused, the rays proceeding from each point of the object being spread over a small circular space of sensible diameter.

Now every hour's experience proves that the eye is capable of seeing distinctly objects whose distances vary within very wide limits. To every eye there is a certain distance at which it is able to see distinctly without any effort; this is called the *distance of perfect indolent vision.* Beyond this distance no internal effort whatever will enable the eye to see distinctly; but within that distance it is capable of discerning objects with perfect distinctness up to a certain limit, which is called the *least distance of distinct vision.* The least distance of distinct vision varies in common eyes from about six to eight inches. The greatest distance, or the distance of perfect indolent vision, varies within much wider limits, and cannot be so readily ascertained; for the generality of eyes, it is so considerable, that they are usually considered fitted to bring *parallel* rays to a focus on the retina.

The question then arises, by what changes of conformation in the eye does it thus become adapted to near distances? Is it by a change in the *position* of the *crystalline lens?* or by a change of the *curvature* of its surfaces, or of that of the *cornea ?* or, lastly, is it produced by a change in the configuration of the entire eye, and an *elongation of its axis ?*

The most plausible hypothesis is that which combines a change of curvature of the *cornea* with a change in the figure of the *entire eye;* an opinion which has been advocated by Ramsden and Sir Everard Home. According to these authors, the *recti*, or straight muscles which move the eye in its orbit, by their simultaneous contraction produce a lateral pressure upon the eye, which both elongates it in the direction of its axis, and, through the medium of the fluids within, forces out the cornea and renders it more convex. This hypothesis, however, has been overturned by an ingenious experiment of Dr. Young, by which it appears that the eye, *when in contact with water*, possesses a power of adaptation to different distances within the usual limits ; a fact which proves that the accommodating power does *not* reside in the *cornea*, the refraction of which, in this case, is annulled by contact with fluids of equal density on both sides.

According to Dr. Young, the accommodating power resides in the *crystalline*, which he supposes to be susceptible of a change of figure, becoming more convex when the eye is directed to near objects, and again recovering its former shape when distant objects are viewed. The regular fibrous structure of the crystalline, already mentioned, seems to indicate a muscular power, for which we can hardly suppose any office, if it be not employed in some adaptation of this nature. And the numerous experiments of Dr. Young, upon persons who had been couched for the cataract, seem to leave little doubt that the principal part, if not the whole, of the adaptation in question must be the work of the crystalline; persons deprived of that lens being found, in a great degree, deficient in the power of seeing distinctly at different distances.

It may be observed, that a change in the aperture of the *pupil* tends, as far as it goes, to adapt the eye to different distances; the diminution of that aperture limiting the breadth of the cone of rays, which converge to a point beyond the retina, and therefore diminishing the space over which they are diffused there. This change, however, though it may conspire in producing the effect in question, is not sufficiently considerable to account for it altogether; and besides, in these variations, the pupil seems to be more affected by the quantity of light than by the distance of the object from the eye.

112. The adaptation of the eye to different distances is however limited; and when the curvature of the surfaces, and therefore the power of the eye in its natural state, is too great or too small, it cannot be adapted to different distances without artificial assistance. Thus, in *short-sighted* persons the curvature of the surfaces of the eye is *too great*, and all rays, excepting those proceeding from a near distance, are brought to a focus *before* they reach the retina; such persons, therefore, can only see near objects distinctly. The defect is remedied, and the person enabled to see remote objects, by the aid of a *concave* lens, the effect of which is to increase the divergence of the incident rays, and, therefore, fit them to be brought to a focus on the retina. In *long-sighted* persons, on the contrary, the curvature of the surfaces of the eye is *too small*, and all pencils, excepting those which proceed from very distant objects, converge to a focus *behind* the retina. The vision of such persons is adapted to near objects by the use of a *convex* lens, which diminishes the divergence, or even gives a convergence to the incident rays, and thereby enables the eye to bring them to a focus on the retina. This defect of vision is common among aged persons, the cornea becoming flatter with age, and therefore the refraction of the eye diminished.

These defects of vision are often brought on by habit. Thus, persons who are accustomed to look much at near objects, as engravers, are apt to become short-sighted; and, on the other hand, those whose attention is generally fixed on distant objects, as savages, who spend the most of their time in the pursuit of game, become long-sighted, and are unable to see near objects distinctly. This is easily explained. The eye and its attendant mechanism being frequently used in one particular state, become gradually fixed in it, and the power of alteration is lost by disuse.

113. The eye, however, though the chief organ of vision, does not constitute the whole of the material apparatus by which it is performed. The impression produced by the images on the retina is communicated to the *optic nerve*, and by means of that to the *brain*, where, by the laws of our compound being, it becomes the condition on which the sensation of vision is produced in the mind. That these several steps of the process are equally essential to vision is fully established by experience. Thus, if through want of transparency in the humours of the eye, or any other cause, the image on the retina is either imperfectly formed or wholly wanting, the vision will be in proportion defective. This is the case in the disease called the *cataract*, in which the crystalline lens loses its transparency, and, therefore, stops either the whole or part of the light incident upon it; the image on the retina is thus either wholly effaced, or becomes extremely indistinct, and blindness, either total or partial, ensues. Again, the humours of the eye may perform their functions perfectly, and an accurate image be depicted on the retina; and yet, through the insensibility of the optic nerve, or some other cause, the impression produced on the retina may not be communicated to the brain, and thus no vision produced. This is the case in a paralysis of the optic nerve, which produces, while it lasts, total blindness.

114. What has been just said will enable us to give an answer to the question on which much absurd discussion has been raised,—namely, why objects appear *erect*, though their images on the retina are inverted? The question, in fact, assumes that these images in themselves *constitute* vision; whereas, all that we know about them in reference to vision is simply this, that the rays diverging from the several points of the object are made to converge to corresponding points in the retina, and there excite impressions which are communicated, by means of the optic nerve, to the brain; and that the perception thence arising has a regular but inexplicable correspondence with the manner in which these rays converge on the retina, or, in other words, with the images formed there.

But further; it is now well ascertained that these perceptions of vision do not, *directly*, give us any idea of the *position* of external objects. It is true the several changes in the latter have corresponding perceptions annexed, by which we learn to judge of them; but it is by *experience* only that we come to know this regular correspondence, and learn to conclude, that what affects our sight in such and such a manner shall also affect our touch in a corresponding manner. This association is formed at so early a period of life, that all traces of its formation are obliterated; and we conclude that to be an *original* perception of *sight*, which is only an *associated* one of *touch*. From all this it manifestly follows that the only thing necessary to enable us to judge of the position of external objects by sight is, that there should be a steady correspondence between these positions and the impressions which the rays proceeding from them produce on the retina.

On the same principles we may solve the question, why, as there is an image formed in each eye, we do not see two objects instead of one.

## CHAPTER IX.

### OF VISION THROUGH A SINGLE LENS.

115. The *distinctness* of objects seen by the naked eye depends chiefly on the accurate convergence of the rays of each pencil to as many distinct points on the retina; and the eye, we have seen (111), is furnished with an organization by means of which it can accommodate itself to the distinct vision of objects whose distances vary within very wide limits. When, however, the refracting surfaces of the eye are too much or too little curved, this power is limited on the one side or the other; the refraction of the eye in the former case being too great for the distinct vision of distant objects, the divergence of the rays proceeding from which is very small; and in the latter too small for the distinct vision of near objects, from which the divergence is considerable. These defects, it was soon seen, might be remedied by the use of a concave or convex lens; and accordingly this application of lenses was made long previous to the invention of the more complicated dioptrical combinations.

The theory of spectacle-glasses is as simple as their construction. It is evident that the distance at which we *desire to see* distinctly by the aid of a lens, and that at which we *actually see* by the naked eye without effort, must be the conjugate focal distances of the lens before and after refraction respectively. Hence, if these distances be denoted by $u$ and $u'$, and the focal length of the lens by $f$, we have

$$\frac{1}{f} = \frac{1}{u'} - \frac{1}{u};$$

by which the focal length of the lens required is known.

For *short-sighted* persons, who desire to see distinctly at a considerable distance, $\frac{1}{u} = 0$, very nearly, and therefore

$$f = u'.$$

The lens employed, therefore, must be a *concave* lens, whose focal length is equal to the distance at which such persons can see distinctly with the naked eye without effort.

In the case of *long-sighted* persons, on the contrary, the distance $u'$, at which they can see distinctly with the naked eye, is very great; so that $\frac{1}{u'} = 0$, very nearly, and therefore

$$f = -u.$$

That is, the lens must be *convex*, and its focal length equal to the distance at which the person desires to see by its assistance.

116. Required the visual angle under which an object is seen through a lens.

Let $a$ and $a'$ denote the linear magnitudes of the object and its image; $u$ and $u'$ their distances from the lens; then (75) $a' = a\,\frac{u'}{u}$. But if $\delta$ denote the distance of the image from the eye, the visual angle under which it is seen is $\frac{a'}{\delta}$, or

$$\frac{a}{\delta}\,\frac{u'}{u}.$$

This is usually compared with the greatest angle under which the object may be seen distinctly by the naked eye, or with $\frac{a}{\lambda}$, $\lambda$ being the *least distance of distinct vision;* and the ratio of these angles is the measure of the magnifying power. It is accordingly equal to

$$\frac{\lambda}{\delta}\,\frac{u'}{u}.$$

117. The eye is necessarily at the side of the lens remote from the incident light. When, therefore, the image is at the same side, or $u'$ negative, there is no limit to the increase of the visual angle dependent on the position of the eye; for $\delta$ may be made as small as we please. For distinct vision, however, $\delta$ cannot be less than $\lambda$; and when the eye is placed at this distance, the magnifying power becomes

$$\frac{u'}{u}.$$

This ratio may be increased indefinitely, by approaching the object to the principal focus.

The value of $u'$ being negative, the case is that of an object presented to a *convex* lens, at a distance greater that the principal focal length. The image in this case is *real*, and *inverted* (76).

118. When $u'$ is *positive*,—or the image at the side of the lens *towards* the incident light,—the visual angle will be greatest when $\delta = u'$, that is, when the eye is close to the lens; and the magnifying power, in this case, is

$$\frac{\lambda}{u}.$$

This value may be increased indefinitely by diminishing $u$, the distance of the object from the lens. For distinct vision, however, there is a limit to the diminution of $u$, and therefore to the increase of the visual angle. For, since the eye is at the lens, $u'$ cannot be less than the least distance of distinct vision. Accordingly, the greatest value of $\frac{1}{u'}$, is $\frac{1}{\lambda}$; and the greatest value of $\frac{1}{u}\left(=\frac{1}{u'}-\frac{1}{f}\right)$ is $\frac{1}{\lambda}-\frac{1}{f}$. The greatest magnifying power, therefore, is

$$1-\frac{\lambda}{f}.$$

$u'$ being positive, the case is that of an object presented to a concave lens at any distance, or to a convex lens at a distance less than the principal focal length. In the former case, $f$ being positive, the visual angle is less than that under which the object is seen by the naked eye; in the latter it is greater. The object is seen *erect*.

The latter case is that of the common *reading-glass*, which is a convex lens of considerable aperture, whose focal length depends on the degree of magnifying power required. The magnifying power is

$$1+\frac{\lambda}{f}.$$

119. The *apparent brightness* of an object, seen through a lens, is equal to that of the object seen by the naked eye, on the supposition that no light is lost in its passage through the lens.

For the quantity of light incident upon the pupil of the eye, diverging from any point of the image, is to that incident upon the same, diverging from the corresponding point of the object, in the inverse ratio of the squares of the distances of the image and object from the eye. The quantities of light, therefore, incident upon the pupil, diverging from the entire object and image, are as their areas directly, and inversely as the squares of their distances; that is, as the squares of the angles which they subtend to the eye, their areas being as the squares of their linear magnitudes. But the spaces occupied by their images on the retina are in the same ratio; and, therefore, the density of the light in these images, or their apparent brightness, will be the same.

There will always, however, be some light lost in its passage through the lens, so that the brightness of the object seen through it will be somewhat less than that of the same object seen by the naked eye.

In the preceding it is assumed that the pencil of rays diverging from each point of the image is sufficiently broad to *fill the pupil* of the eye. When this is not the case, the brightness will be less in the proportion of the area of the section of the pencil.

120. In looking through spectacles, when the eye is directed to any part of the field remote from the centre, the rays which are incident upon the pupil pass through the lenses very obliquely; and, therefore, the marginal parts of the field will be much distorted and confused. This obliquity, and the consequent confusion, may be in a great degree remedied by making the inner surfaces of the lenses concave; and it is obvious that, the nearer the curvature of these surfaces approaches to that of the cornea, the less will be the obliquity of the marginal pencils. This is the principle of Dr. Wollaston's *periscopic spectacles*, the lenses of which are of the form of the *meniscus*, or the *concavo-convex*, with their concave surface turned towards the eye,—the former being intended for long-sighted, and the latter for short-sighted persons. Where *extent of field* is required, as in the open air, these forms are much to be preferred to the common ones. When, however, the eye is steadily directed to the centre of the field, as in reading, or in the examination of any minute object, they are inferior to the com-

mon forms, inasmuch as the aberration of the central pencils is considerably increased.

121. The great impediment to the vision of minute objects—namely, the smallness of the visual angle under which they are seen—might appear at first sight to be easily removeable, and to any desired extent, by simply diminishing their distance from the eye. To this diminution of distance, however, we have seen, the nature of vision imposes a limit, the eye being incapable of bringing rays to a focus on the retina whose divergence exceeds a certain magnitude. Accordingly, the distance determined by this limit is denominated the *least distance of distinct vision;* and the angle, which any small object subtends to the eye at that distance, is the *greatest angle* under which it can be seen distinctly by the naked eye.

If the object be placed, however, in the focus of a convex lens, the rays of the several pencils, proceeding from the several points of the object, emerge parallel to one another, and to the lines connecting those points with the centre of the lens. Consequently, the angle under which it will appear is equal to that which it would subtend to the naked eye at a distance equal to the focal length of the lens; and the rays of each pencil, being parallel, will be brought to a focus on the retina without effort. Consequently, the object will appear at once distinct, and magnified to the same extent as if we were enabled to approach it to a distance equal to the focal length of the lens. A lens of high power, thus employed, is called a *single microscope.*

122. The visual angle under which the object is seen by the aid of such a lens is

$$\frac{a}{f};$$

$a$ denoting the linear magnitude of the object, and $f$ the focal length of the lens. But the angle which the object would subtend to the naked eye, at the least distance of distinct vision, is $\frac{a}{\lambda}$; and the ratio of these angles, or the magnifying power, is

$$\frac{\lambda}{f}.$$

Accordingly, the magnifying power is increased by diminishing the focal length of the lens. The focal lengths of lenses, fitted up as single microscopes, are usually 1 inch, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{20}$ of an inch. In examining minerals or flowers a high power is not required, and accordingly the focal lengths of the lenses used in the botanical or mineralogical microscope are seldom less than one-fourth of an inch. For various other purposes, however, it is desirable to obtain as high a power as possible; but, when a higher power is sought than that furnished by a lens of $\frac{1}{20}$th of an inch focal length, it is usual to employ, instead of lenses, small spherules of glass; the construction of the latter being easier than that of the former, when the curvature is considerable.

123. The available power of the single microscope seems to be limited by the constitution of the eye. It was observed by Huygens, and has since been fully confirmed by the observations of others, that the vision will be imperfect if the diameter of the cylindrule of rays, which are incident upon the eye emerging from a lens, falls below a certain limit, which he has fixed at the $\frac{1}{72}$ of an inch. But the diameter of this cylindrule is equal to the aperture of the lens or spherule; this aperture then must not be less than the $\frac{1}{72}$ of an inch, even supposing the whole of it to be effective; and, therefore, the focal length of a glass spherule, —which is equal to three-fourths of its diameter —cannot be less than about the $\frac{1}{100}$th of an inch.

Another disadvantage, attendant on the employment of lenses or spherules of small aperture and high power, is the want of brightness. When an object is placed in the focus of a lens or spherule, the breadth of the cylinder of rays, proceeding from each point of the image, is equal to the aperture of the lens; and, when that aperture is less than that of the pupil of the eye, the apparent brightness of the object, seen through the lens, is less than that of the same object seen by the naked eye, in the duplicate ratio of the aperture to that of the pupil. The apparent brightness, therefore, decreases as the square of the aperture; and, when the latter is very small, becomes so inconsiderable as to require the object to be strongly illuminated.

124. The confusion of vision, arising from spherical aberra-

tion, is measured by the area of the circle of aberration in the image on the retina; and the diameter of this circle, it may be shown, varies as the cube of the aperture directly, and inversely as the cube of the focal length, the material of which the lenses are composed, and their form, being supposed the same. For a given aperture and focal length, the best form of a lens of crown glass is that in which the radii of the surfaces are as 1 to 6, the less curved surface being turned towards the object.* The aberration of the plano-convex lens, having its plane surface towards the object, is but little greater; and, as it is much easier of construction, it is the usual form of lenses fitted up for the single microscope. The aberration in such a lens is less than in an equiconvex, in the ratio of 2 to 3 nearly.†

125. The errors of form are of a more various and complicated nature in their effect on those pencils which pass through the lens *excentrically;* such as, in fact, are all the pencils which reach the eye proceeding from any part of the field but its centre. There will be *distortion* of the marginal parts of the field, arising from a difference in the inclination of the emergent pencils to the axis. There will be *indistinctness*, either at the margin or at the centre of the field, resulting from the impossibility of adjusting both to such a distance from the lens, as will insure the parallelism of the emergent pencils. And, finally, from this difference in the pencils proceeding from the centre and from the edges, the field of view will appear to be *convex* instead of flat; the pencils proceeding from the central parts emerging divergent, when those which proceed from the margin emerge parallel.

In lenses of very small aperture these errors may be disregarded. But when the objects to be examined are large, large apertures are required, and the errors become considerable. They may be corrected, severally, by combining two or more lenses of proper forms; but, as the combination which diminishes one source of error frequently enlarges another, it will be simpler and more effective in practice to correct all by diminishing the aperture of the lens, provided such a diminution can be rendered compatible with a sufficient extent of field. This object is attained in the *periscopic lenses* invented by Dr. Wollaston. They are composed of two plano-convex lenses, united together at their plane sides, between which a thin plate of brass is interposed, having a small circular aperture in its centre. In this construction, it is evident, a considerable field of view is preserved, while at the same time all the advantages of diminished aperture, in correcting the aberration both of the central and excentrical pencils, are obtained. The diameter of the aperture which Dr. Wollaston found to be most convenient was one-fifth of the focal length of the compound lens.

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* *Treatise on Light and Vision*, p. 135.

† *Ibid.*, p. 133.

126. A convenient microscope for temporary purposes may be readily constructed by perforating a thin plate of brass, and inserting a drop of water in the aperture. The edges of this aperture are to be made as thin as possible, in order that the fluid may assume the requisite form, which will be that of a double convex lens. An instrument of still simpler construction is suggested by Sir David Brewster, and consists merely of a drop of some transparent fluid, such as Canada balsam or turpentine varnish, upon a thin plate of glass whose surfaces are exactly parallel. A plano-convex lens will thus be formed, whose curvature, and therefore its focal length, will be different according as the drop is above or below the plate. The former of these substances will soon harden, and, if preserved from dust, continue perfect for a considerable time.

# CHAPTER X.

## OF VISION THROUGH A SYSTEM OF LENSES.

127. We have already seen that, when a convex lens is presented to a distant object, an inverted image of that object will be formed near its focus, which will be greater or less in the proportion of the focal length. If then the eye be placed at a short distance beyond this image, it will receive the rays diverging from it, and be affected by them in the same manner as if they proceeded from a real object in the same place; and the angle under which the object will appear will depend on the focal length of the lens, and on the distance of the eye from the image. But if, instead of observing the image with the naked eye, we look at it through a lens of short focus, placed at the distance of its own focal length from it,—just as we would examine any small object with a common magnifier,—the image will not only be seen under a greater angle, and therefore more magnified than before, but also the rays of each pencil, emerging parallel from the second lens, will be adapted to distinct vision.

Such is the principle of the *common astronomical* telescope, the construction of which was first explained by Kepler. It consists, accordingly, of two convex lenses, BC, *bc* (fig. 30), fixed at the extremities of a tube, at a distance equal to the sum of their focal lengths. The lens BC, which is turned towards the object, is called the *object-glass*, and has a large aperture and considerable focal length; that turned towards the eye, *bc*, is called the *eye-glass*, and is of small aperture and short focal distance. An inverted image of the object, *pq*, is formed at the common focus of the two lenses. B*q*, A*q*, and C*q*, are three rays of a pencil which diverge from any one point of the object, and by the refraction of the object-glass are made to converge to $q$, the corresponding point of the image; crossing at this point, they are incident upon the eye-glass diverging, and by its refraction are made to emerge parallel.

Since $A\alpha\epsilon$, the axis of the pencil proceeding from the upper part of the object, crosses the axis of vision again at $\epsilon$, and therefore proceeds to the upper part of the eye, the image on the retina will be *erect* with respect to the object; and, consequently, the object will appear in the opposite position to that which it has to the naked eye, or *inverted.*

128. The magnifying power of the telescope is determined by the ratio of the angles, $\alpha Aa$, $\alpha\epsilon a$, which the axis of the pencil, $A\alpha\epsilon$, makes with the axis of the lenses before and after refraction by the eye-glass, respectively; for it is manifest that $\alpha Aa$, or $qAp$, is the angle which the portion of the object, whose image is $pq$, subtends at the centre of the object-glass, or at the eye, $q.p$; and that $\alpha\epsilon a$ is the angle under which it is seen by the telescope. But if, from the point $q$, the line $qa$ be drawn to the centre of the eye-glass, $\alpha\epsilon$ is parallel to $qa$ (121), and consequently $\alpha\epsilon a = qap$. Accordingly, the magnifying power is represented by the quotient $\frac{qap}{qAp}$, or by $\frac{Ap}{ap}$, very nearly. Now $Ap$ is the focal length of the object-glass, the object being supposed infinitely distant, and $ap$ is the focal length of the eye-glass. Wherefore, denoting them by $F$ and $f$, respectively, the magnifying power is

$$\frac{F}{f}.$$

When the object cannot be regarded as infinitely distant, $Ap$ is the conjugate focal distance after refraction by the object-glass, and it is equal to $\frac{uF}{u-F}$ (70), $u$ being the distance of the object. Hence the magnifying power is

$$\frac{F}{f}\cdot\frac{u}{u-F},$$

and is increased by the adaptation to near objects, in the ratio of

$u$ to $u - F$. This adaptation is effected by increasing the distance of the eye-glass from the object-glass.

129. The causes which limit the vision of distant objects by the naked eye,—namely, the want of sufficient light, and the smallness of the visual angle under which their parts appear,—are both remedied by this instrument. The quantity of light incident upon a given surface decreases as the square of the distance increases; and, the magnitude of the pupil being nearly the same under all circumstances, it follows that the quantity of the light which enters the eye from a distant object, and therefore the impression made upon the sense of vision, decreases in the same ratio. Now the object-glass of the telescope collects a quantity of light as much greater than that received by the naked eye as the area of the object-glass exceeds that of the pupil; and though a certain portion of this is absorbed in its passage through the lenses, or lost by reflexion at their surfaces, yet on the whole the quantity is much increased.

The other impediment to the distinct vision of distant objects is the smallness of the visual angle under which their parts appear. It is found by experience that no portion of a remote object can be distinctly seen by the naked eye, when it subtends there a smaller angle than about one minute. But our vision, which is thus naturally limited, may be extended indefinitely by means of the telescope; for we have seen that the angle under which an object is seen, when viewed through a telescope, exceeds that under which it appears to the naked eye, in the ratio of the focal length of the object-glass to that of the eye-glass.

130. If the eye-glass be *concave* instead of convex, and be placed *between* the object-glass and its principal focus, at a distance from the latter equal to its own focal length, the rays of each pencil fall upon it converging to the principal focus of rays proceeding in an opposite direction, and consequently emerge parallel.

This is the construction of the telescope, which has been named after its inventor, Galileo. It is represented in fig. 31, in which BC represents the object-glass, and *bc* the eye-glass.

*pq* is an inverted image of the object, formed in the common focus of the two lenses. B*q*, A*q*, and C*q*, are three rays of a pencil which proceed from any one point of the object, and by the refraction of the object-glass are made to converge to *q*, the corresponding point of the image; these rays, being incident upon the eye-glass converging to the principal focus of rays proceeding in the contrary direction, are by its refraction made to emerge parallel to one another, and to *aq*, the axis of the pencil.

Since $\alpha\epsilon$ is parallel to *aq*, *qap* is the angle under which the object, whose image is *pq*, is seen by the telescope; and *q*A*p* being the angle under which it is seen by the naked eye, the magnifying power is equal to $\frac{\mathrm{A}p}{ap}$, the ratio of the focal lengths of the object-glass and eye-glass, as in the astronomical telescope.

It will be seen from the figure that the rays which diverge from the uppermost part of the object proceed to the lowermost part of the retina, and *vice versâ;* and therefore the object is seen in the same position as by the naked eye. Accordingly, objects seen through the Galilean telescope appear *erect*, instead of being inverted, as in the astronomical telescope. On account of this advantage, as well as because it is shorter than an astronomical telescope of the same power (its length being the *difference* of the focal lengths of the two lenses, while in the astronomical it is their *sum*), the Galilean telescope is generally preferred in instruments of low power. The common opera-glass, which magnifies about three or four times, is always of this construction.

131. The angular extent of any object, seen through the astronomical telescope, is defined by the axes of the extreme pencils which fall on the eye-glass, being equal to the angle contained by these axes, *b*A*c* (fig. 32). This angle is denominated the *field of view;* and it is evidently equal to the angle which the diameter of the eye-glass subtends at the centre of the object-glass.

But in order to take in the whole extent of this field, the eye must be placed at the point in which the axes of the extreme pencils intersect, after refraction by the eye-glass. Let A*b* be one of these axes, drawn from the centre of the object-glass to the edge of the eye-glass, and let it intersect the perpendicular raised at the common focus of the lenses in $q$. Then, if from $q$ the line $qa$ be drawn to the centre of the eye-glass, the ray $Ab$ will be refracted in a direction, $bd$, parallel to $qa$. Hence the triangles, $Aqa$, $Abd$, are similar, and we have $Aq : Ab :: qa : bd$, or $Ap : Aa :: pa : ad$. That is, the distance of the eye from the eye-glass is a fourth proportional to the focal length of the object-glass, the focal length of the eye-glass, and the sum of these focal lengths. In all telescopes there is a diaphragm with a small aperture, called an *eye-stop*, placed at this distance from the eye-glass, in order to insure to the eye the required position.

In the Galilean telescope there is no point corresponding to that we have been describing; for the axes of the several pencils diverge, after refraction by the eye-glass. In this case, then, the eye must be placed as close as possible to the eye-glass, in order that it should take in the widest extent of field. It will be easily seen that the visible field is limited by the lines drawn from the centre of the object-glass to the edge of the pupil; or that the field of view is equal to the angle which the diameter of the pupil subtends at the centre of the object-glass. The aperture of the pupil being much less than that of the eye-glass, the field in the Galilean telescope is much more contracted than in the astronomical.

132. The *quantity of light* which enters the eye is proportional to the quantity which falls on the object-glass; and this is evidently proportional to the area of the object-glass, or to the square of its aperture. Now it is found that the power of discerning minute objects in the starry concave, or, as it has been called, the *space-penetrating power*, depends on the quantity of light. This power, therefore, is measured by the square of the aperture of the object-glass.

The *brightness* of an object seen through a telescope depends, not on the quantity of light alone, but also on its apparent magnitude. It is, in fact, measured by the intensity of the light in the image on the retina; and this is as the quantity of light directly, and inversely as the space over which it is diffused. But the quantity of light, we have seen, varies as $A^2$, $A$ being the aperture of the object-glass; and the space over which it is diffused, or the area of the image, varies as the square of the magnifying power, or as $\left(\frac{F}{f}\right)^2$. Hence the brightness is proportional to

$$\left(A\,\frac{f}{F}\right)^2.$$

When, therefore, we desire much brightness in a telescope, we must use a large object-glass, and apply a low magnifying power. Such is the construction of the common night-glass, an instrument much used at sea.

133. The *indistinctness* of an object, seen through a refracting telescope with a single object-glass, is due, mainly, to the chromatic dispersion of the light in its passage through that lens. It has been shown (100) that, owing to this cause, the rays diverging from a point, and incident upon a lens, are not made to converge to a point after refraction; and that the least space into which they are collected is a small circle, called the circle of chromatic aberration. Hence, for each *point* of the object, there will be a corresponding *circle* in the image formed by the object-glass; and the overlapping of these circles will produce confusion in the image. This confusion will be magnified by the eye-glass; and its resulting value will depend on the angle which the diameter of the circle of chromatic aberration subtends at the eye-glass. But, when the object is very distant, the diameter of the circle of chromatic aberration is equal to $\pi A$, $\pi$ being the dispersive power of the material of which the object-glass is composed (101); and the angle which this subtends at the eye-glass is equal to $\frac{\pi A}{f}$. The confusion varies as the square of this quantity, or as

$$\left(\frac{\pi A}{f}\right)^2.$$

134. The magnifying power of the common astronomical and

Galilean telescopes may be increased either by increasing the focal length of the object-glass, or diminishing that of the eye-glass. But it is evident that, in an instrument such as has been described, the latter method cannot be employed to any extent; for, in proportion as we diminish the focal length of the eye-glass, we increase at the same time the confusion. Consequently, to attain a high magnifying power, and to preserve at the same time the distinctness of the instrument, it became necessary to increase greatly the focal length of the object-glass, and therefore, also, the length of the instrument. Tubes of great length, however, were liable to bend, and were found to be in other respects extremely unmanageable. Huygens, accordingly, dispensed with them altogether, and fixed the object-glass to the top of a long pole, with an apparatus attached by which its position could be altered at pleasure. These instruments were called *aerial* telescopes. Huygens's great telescope was 123 feet in length; and the length of that used by the astronomer Cassini was 150 feet.

Accordingly, the first great step in the improvement of the refracting telescope is to be dated from the discovery of the principle of achromatism. It has been shown that two lenses may be combined, which shall correct each other's dispersions, while, nevertheless, the refraction of one of them predominates, and the rays are brought to a focus (103). If then such a compound lens be employed as the object-glass of a telescope, there will be no chromatic aberration, and therefore no confusion in the image thence arising. The magnifying power of the telescope may therefore be increased by diminishing the focal length of the eye-glass, instead of increasing that of the object-glass, and the instrument is thus rendered convenient and manageable.

135. It remains to consider the errors of the *eye-glass*, and its various forms.

It is evident that the eye-glass has its imperfections as well as the object-glass. These are twofold,—viz., the errors due to the *form* of the refracting surfaces, and those which arise from the unequal refrangibility of the rays of light. We have overlooked the errors of the former class, in the case of the object-glass, as of comparatively small importance. The case of the eye-glass is, however, different. The axes of the several pencils incident upon the object-glass all pass *through its centre*, and generally at very *small obliquity;* while all those which fall upon the eye-glass (excepting those proceeding from the middle of the field) intersect the eye-glass *excentrically*, and with *considerable obliquity*. The spherical aberration of the eye-glass is, accordingly, more complicated in its nature than that of the object-glass, and more injurious in its effects.

These effects may be classed under three heads:

I. The first is a *distortion* of the object. The axes of the extreme pencils proceeding from the centre of the object-glass will, by the refraction of the eye-glass, meet the axis of the telescope at a nearer point than those of the central pencils. The ratio of the visual angles, therefore, will be greater in the extreme parts of the field than at its central parts; these parts, accordingly, will be unduly enlarged, and the object will appear distorted.

II. A second effect is *indistinctness*. The image of a distant object, formed in the focus of the object-glass, is nearly a spherical surface whose centre is the centre of the object-glass. It is therefore *convex* towards the eye-glass, instead of being concave towards it, as it should be in order that all its parts may be in focus. Hence, when the middle of the image is at the proper distance from the eye-glass, the extreme parts will be too distant; so that if the centre of the field be seen distinctly, the margin will be indistinct; and conversely. This defect increases with the magnitude of the field.

III. Lastly, the rays of a small pencil diverging from a point, and incident upon a lens *excentrically*, are not brought by refraction anywhere to a point,—the rays in the plane passing through the axis of the pencil and the axis of the lens converging, in general, more rapidly than those in the perpendicular plane. It may be shown, accordingly, that the image of a point formed by such a pencil on the retina is an *ellipse*, the ratio of whose axes is different under different circumstances.

These effects of spherical aberration in the eye-glass can only be diminished by the employment of two or more lenses, instead of one; and the resulting aberration will be least, *cæt. par.*, when the whole refraction, or bending of the ray, is *equally divided* among the lenses. In the astronomical telescope the eye-lens is always *double*, and in such an eye-piece the condition of equal refraction will be satisfied, when the *interval of the lenses is equal to the difference of their focal lengths.*[*]

136. The errors of *chromatic aberration*, likewise, assume a different form in the object-glass and in the eye-glass, and for the reasons already referred to. When a pencil of rays falls upon a lens *excentrically*, as is generally the case with the eye-glass, the edge of the lens acts as a prism, and any single white ray of the pencil will be dispersed into its coloured elements; and as these different species of simple light intersect the axis of the lens at different angles, the edges of the object from which they proceed will appear bordered with *coloured fringes.* This confusion increases with the visual angle; and, towards the margin of a wide field, it is of such magnitude as altogether to mar the appearance of the object.

This effect of chromatic aberration is manifestly different from the error of the object-glass derived from the same source; and, accordingly, the mode of correcting it is also different. In fact, the chromatic aberration of the eye-glass may be corrected by two lenses of the same material, and even of the same form, by simply placing them at a proper interval. When the lenses are of the same material, as is usually the case, theory shows that the interval between them, which will render the combination achromatic, is *half the sum of their focal lengths.*[†]

137. Combining the conditions of equal refraction and achromatism, given in the preceding articles,

$$f-f'=e=\tfrac{1}{2}(f+f');$$

$f$ and $f'$ being the focal lengths of the two lenses, and $e$ the interval between them; whence

$$f=3f', \qquad e=2f'.$$

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[*] *Treatise on Light and Vision*, p. 337.

[†] *Ibid.*, p. 340.

That is, the focal length of the first lens must be triple that of the second, and the interval between them double of the same.

This is the construction of the Huygenian eye-piece. The two lenses, A and B (fig. 33), are usually plano-convex lenses, having their plane surfaces next the eye; the first of them is generally called the *field-glass*, and the second the eye-glass. The rays which by the refraction of the object-glass converge to the image $p'q'$ in its focus, are, by the refraction of the field-glass, made to converge to the image $pq$, which is in the focus of the eye-glass, and therefore emerge parallel.

138. This construction cannot be employed in telescopes having a micrometer in their focus for the measurement of angles. For the image, being formed *between* the two lenses, will be distorted by the refraction of the field-glass, and, therefore, equal divisions of the micrometer will not correspond to equal angles. In such telescopes the image must be *before* the field-glass, in which arrangement, though the image is distorted, yet, the micrometer wires being equally distorted, no error will result in the measurement.

Such is the construction of the *astronomical eye-piece*. The two lenses in this eye-piece are usually of equal focal lengths, and, therefore, the condition of achromatism requires that the interval between them should be equal to that focal length. But, in this arrangement, the field-glass being exactly in the focus of the eye-glass, any dust which might happen to lie upon it, or any flaw in the glass itself, would be distinctly seen through the eye-glass, and mar the vision of the object. The interval of the lenses is, for this reason, made *two-thirds* of the focal length of either; and thus, though the condition of achromatism is not satisfied, yet the departure from it is not considerable. This eye-piece is represented in fig. 34. in which A and B represent the field-glass and eye-glass respectively; $pq$ and $p'q'$ the images from which the rays diverge before and after refraction by the former. It is easily seen that $Ap$, the distance of the field-glass from the focus of the object-glass, is one-fourth of its own focal length.

The lenses are usually *plano-convex* lenses of equal curvatures, with their curved surfaces turned towards each other. The indistinctness arising from spherical aberration in this eye-piece is much less than in any other of the ordinary constructions.

139. The *inversion* of the object, which takes place in the astronomical telescope, is of little consequence as long as its use is thus limited. In telescopes intended for *terrestrial* observations, however, it is absolutely necessary that the objects should be represented in their natural position; and we have next to consider the forms of the eye-piece by which this is attained.

The *terrestrial eye-piece*, invented by Rheita, consists of *three* lenses, the distance of the first of which from the object-glass is equal to the sum of their focal lengths, while the interval of the second and third lenses is also equal to the sum of their focal lengths; in other words, it is a common astronomical telescope, with two additional lenses placed at an interval equal to the sum of their focal lengths. It is represented in fig. 35, in which A, B, and C, are the three lenses; *qabcd* a pencil of rays proceeding from any point of the object, and refracted by the object-glass. This pencil is brought to a focus at *q* and *s*, where two images of the object, *pq* and *rs*, are formed, the former in the common focus of the object-glass and the lens A, and the latter in the common focus of the lenses B and C. Between A and B the rays of each pencil are parallel. As it is usually constructed, the apertures and the focal lengths of the three lenses are equal, and the intervals between them are each equal to double of the common focal length.

The three-glass eye-piece, in the disposition described, is not achromatic. It is easy, however, to render it so, by simply altering the intervals of the lenses. The focal lengths being supposed equal, we have only to increase the interval of the second and third lenses to $2\frac{1}{2}$ times the common focal length.* In this construction, also, the bending of the ray is equally divided among the three lenses; and the magnifying power is the same as before.

In the three-glass eye-piece, the refraction at each lens is considerable, since the extreme pencil must intersect the axis twice. It has been accordingly laid aside for one with *four* glasses, which is now the usual form of the eye-piece for terrestrial telescopes. The construction of this eye-piece will be understood from fig. 36, in which A, B, C, D, represent the lenses taken in their order; *abcde* the course of the extreme pencil, intersecting the axis of the telescope between the first and second lenses, and a second time after refraction by the last. The rays of this pencil, diverging from the point $q$ in the focus of the object-glass, are again brought to a focus at $s$, between the third and fourth lenses; and thus a second image, $rs$, is formed, which is in the focus of the lens next the eye. The whole of the light passes through a small circle at each of the points in which the axis of the extreme pencil intersects the axis of the lenses; and it is usual to place, at every such point, a plate perforated with a corresponding aperture, in order to stop erratic light.

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* *Treatise on Light and Vision*, p. 349.

140. The *compound microscope* may be considered as an astronomical telescope adapted to near objects. The object-glass is one of very short focal length, and small aperture; and the object being placed a little beyond its focus, an inverted and magnified image will be formed at the other side of the lens, and at a distance which is considerable as compared with that of the object. This image is viewed, as in the astronomical telescope, through an eye-glass placed at a distance from it equal to its own focal length; and thus a second amplification takes place, with all the circumstances necessary to distinct vision.

The interval of the lenses is usually fixed in this instrument, and the adjustment to distinct vision is effected by varying the distance of the object from the object-glass. For this purpose the object is placed on a stage; and the tube containing the lenses, or the stage itself, is moved by means of a rack and pinion.

141. Let $F$ and $f$ denote the focal lengths of the object-glass and eye-glass. Then, $a$ and $a'$ being the linear magnitudes of the object and its image, the angle under which the latter is viewed in the microscope is $\frac{a'}{f}$; and, the angle which the former subtends to the naked eye, at the least distance of distinct vision, being $\frac{a}{\lambda}$, the

*magnifying power* is equal to $\frac{a'}{a}\frac{\lambda}{f}$. But, $u$ and $u'$ being the distances of the object and image from the object-glass,

$$\frac{a'}{a} = \frac{u'}{u} = \frac{u'+F}{F};$$

but, $e$ being the interval of the lenses, $u' = e - f$, and the numerator of this fraction is $e + F - f$ or $e$, *q. p.*, since $F$ and $f$ are small in comparison with $e$. Accordingly, the magnifying power is, nearly,

$$M = \frac{\lambda e}{Ff}.$$

The *field of view* is limited by the axes of the extreme pencils, which pass through the centre of the object-glass and are transmitted through the eye-glass; and it is accordingly equal (as in the astronomical telescope) to the angle which the diameter of the eye-glass subtends at the centre of the object-glass.

142. The *quantity of light* in the image of an object, seen through the compound microscope, is proportional to that which falls on the object-glass, and, therefore, varies as the square of its aperture directly, and inversely as the square of the distance of the object from it. Hence, $A$ denoting the linear aperture, the quantity of light is proportional to

$$\left(\frac{A}{u}\right)^2;$$

and, therefore, varies in the duplicate ratio of the angle which the diameter of the object-glass subtends at the object.

The *apparent brightness* is measured by the quantity of light in the image on the retina, divided by the space over which it is diffused. But the area of the image on the retina is equal to $M^2$, that of the image on the retina of the naked eye being unity. Hence the brightness is measured by

$$\left(\frac{A}{Mu}\right)^2.$$

143. The object-glass of the compound microscope, when single,

is usually a *plano-convex* lens, having its plane side next the object; that being nearly the form in which the aberration of figure is the least (124). The confusion arising from chromatic aberration is removed, as in the telescope, by employing a *compound object-glass*, consisting of two or more lenses in contact, of different refractive and dispersive powers. The condition of achromatism in such a lens is the same as in the object-glass of the achromatic telescope, being independent of the distance of the radiant point when the lenses composing the system are in contact (103).

The compound object-glass is usually *triple*, being composed of a concave lens of *flint glass* between two convex lenses, one of which is of *crown glass* and the other of *Dutch plate*. These object-glasses sustain an aperture equal to half their focal lengths; whereas the aperture of the single object-glass cannot be much greater than one-eighth of its focal length. Thus the quantity of light, as well as the distinctness, is much augmented.

The eye-glass, in all good microscopes, is also compound, and is generally of the Huygenian form.

# CHAPTER XI.

## OF VISION BY MIRRORS AND LENSES COMBINED.

144. If, instead of a convex lens, a concave reflecting speculum be employed to receive the rays proceeding from a distant object, an image will be formed at its focus, which, if the aperture of the speculum be sufficiently large, may be viewed directly through an eye-glass placed at the distance of its own focal length from the image, as in the common astronomical or Galilean telescopes.

Such is the principle of the reflecting telescope in its simplest form, and was that adopted by Sir William Herschel in the construction of his great instruments. In order that the head of the spectator may intercept as little light as possible, the axis of the speculum must be slightly inclined to the direction of the incident rays, and thus the image thrown near the edge of the tube, where it is viewed through the eye-glass by the spectator having his back towards the object. This method of observation is called the *front view.*

The celebrated instrument, erected at Slough by Sir W. Herschel, in the year 1789, was, until recently, the largest reflecting telescope ever made. The tube was thirty-nine feet four inches in length, and four feet ten inches in diameter. The diameter of the metallic speculum was 49½ inches, and that of its polished surface 48 inches; its thickness was 3½ inches; and its weight, when it came from the furnace, 2118 pounds.

145. The angle under which the object is thus seen is equal to $\frac{i}{f}$, $i$ denoting the linear magnitude of the image formed by the speculum, and $f$ the focal length of the eye-glass. But the angle which the object subtends to the naked eye, or at the centre of

the speculum, $q.p.$, is equal to that subtended by its image at the same place, or equal to $\frac{i}{F}$, $F$ denoting the focal length of the object-speculum. Wherefore the ratio of these angles, or the *magnifying power*, is, as in the common astronomical telescope,

$$M=\frac{F}{f}.$$

It is evident that an object seen through such an instrument will appear inverted.

146. The *field of view* in this telescope, determined by the axes of the extreme pencils, is equal to the angle which the aperture of the eye-glass subtends at the centre of the speculum, or at a distance equal to the difference of the focal lengths of the speculum and eye-glass; and this, since the focal length of the eye-glass is very small in comparison with that of the object-speculum, is very nearly equal to the angle which the aperture of the eye-glass subtends at the vertex of the speculum.

On account of the small extent of field in these instruments, and the consequent difficulty of discovering the object, it is usual to attach, at the side of the tube, a small refracting telescope of low power and considerable field, whose axis is exactly parallel to that of the telescope to which it is joined. The object being found by means of this telescope, it is evident that the larger instrument will be in the desired position. Such an accessory is called a *finder*.

147. The principle of the *front view*, it is evident, is wholly inapplicable in instruments of moderate dimensions. In the telescope proposed and constructed by Sir Isaac Newton, the rays reflected by the object-speculum are received upon a small plane mirror placed between the object-speculum and its principal focus, and inclined at an angle of 45° to the axis of the telescope. By this the rays, which tend to form an image at the focus of the object-speculum, are reflected laterally; and thus an image is formed near the side of the tube, equal and similar to the former. This image, whose plane is parallel to the axis of the telescope,

is viewed through an eye-glass placed at the side of the instrument, at a distance from it equal to its own focal length.

This construction is represented in fig. 37, in which AB is the object-speculum, A′B′ the plane speculum, and *ab* the eye-glass. BB′*gbe* represents part of a pencil of rays proceeding from any point of a distant object. These rays, which by the reflexion of the object-speculum are made to converge to G, the corresponding point of the image FG, at its principal focus, are intercepted by the plane mirror A′B′, and made to converge to *g*, where an image *fg* will be formed, equal to FG. The rays, then crossing at *g*, are incident diverging upon the eye-glass *ab*, placed at a distance from the image equal to its own focal length; and finally, emerging parallel, are received by the eye at *e*, the intersection of the axis of the pencil with the axis of the lens.

In order that the brightness of the central part of the field may be as great as possible, the plane mirror must be of such form and dimensions as exactly to receive the whole of the principal pencil, or the cone of rays converging to the principal focus of the object-speculum. The mirror, accordingly, must be a section of that cone, formed by a plane inclined at an angle of 45° to its axis; and must, therefore, be an *ellipse*, whose axes are in the ratio of $\sqrt{2}$ to 1, very nearly.

148. Since $A'f = A'F$, $AA' + A'f = AF$, AF being the focal length of the object-speculum. Hence the sum of the distances of the object-speculum and eye-glass from the plane reflector, $AA' + A'a$, is equal to the sum of their focal lengths.

The magnifying power of the Newtonian telescope is evidently the same as in that of Sir William Herschel; and is equal to $\frac{F}{f}$, the ratio of the focal lengths of the object-speculum and eye-glass.

149. If a real object be supposed to occupy the place of the image *fg* (fig. 37), it is evident that the rays of each pencil, after reflexion by the plane speculum A′B′, will fall upon the concave speculum diverging from the image FG in its principal focus; and

will, therefore, be reflected parallel. But if the object be removed ever so little farther from the plane speculum, the rays, after reflexion by the two specula, will converge to an image in the axis of the concave; and this image may be viewed through an eye-glass placed at a distance from it equal to its focal length.

Such is the principle of the *reflecting microscope*, invented by Professor Amici. In order to avoid the errors of figure, the concave mirror is an *ellipsoid* of revolution round the greater axis, whose foci are the places of the conjugate images. The microscopic object is placed a little without the tube, on a small shelf projecting from the stand of the instrument, and is illuminated by concave mirrors.

150. The invention of the reflecting telescope is generally ascribed to James Gregory, who described the instrument now called by his name in the year 1663. The Gregorian telescope consists of two concave specula disposed along the same axis, with their concave surfaces facing each other, and at an interval a little greater than the sum of their focal lengths. In the vertex of the larger, or object-speculum, is a circular aperture, to which is attached a tube containing the eye-glass. Now when the axis of the telescope is turned towards any distant object, an image is formed at the focus of the object-speculum. The rays diverging from this image are received by the small speculum, which is at a distance from it a little greater than its own focal length; and by its reflexion a second image is formed near the vertex of the object-speculum, which is viewed through the eye-glass placed there, at a distance from it equal to its own focal length.

This construction is represented in fig. 38, in which AB is the object-speculum, A′B′ the small speculum, and *ab* the eye-glass. BGB′*gbe* represents part of a pencil of rays proceeding from any point of a distant object. These rays are, by the reflexion of the object-speculum, made to converge to G, the corresponding point of the image FG, formed at its principal focus. The distance of this image from the small speculum, A′F, being a little greater than its focal length A′F′, the rays diverging from G, and incident upon the small speculum, will be reflected to *g*, where a second image, $fg$, will be formed at a distance, $A'f$, conjugate to $A'F$. This second image being in the focus of the eye-glass $ab$, the rays diverging from $g$, and incident upon the eye-glass, are, by its refraction, made to emerge parallel, and are received by the eye at $e$, the intersection of the axis of the pencil with the axis of the instrument.

For common purposes the Gregorian telescope is generally preferred to the Newtonian. Its superiority seems to arise from this, that the two specula may be so matched that the irregularities in their figure shall compensate one another in their effects upon the images; whereas in the Newtonian there is nothing to counteract any defect of form of the object-speculum.

151. Since the points F and $f$ are conjugate foci to the small speculum, $\frac{1}{A'F}+\frac{1}{A'f}=\frac{1}{A'F'}$, F′ being its principal focus (33). But $A'F = A'A - AF$; and $A'f = A'a - af$. Wherefore, if $F$ and $f$ denote the focal lengths of the object-speculum and eye-glass, $e$ and $e'$ their distances from the small speculum, and $F'$ the focal length of the latter, there is

$$\frac{1}{e-F}+\frac{1}{e'-f}=\frac{1}{F'}.$$

This is the condition to be fulfilled by the focal lengths and intervals, in order that there may be distinct vision. The eye-glass is usually at a *fixed* distance from the small speculum, and the adjustment to distinct vision is effected by altering $e$, the distance of the latter from the object-speculum.

152. The angle under which an object is seen in the Gregorian telescope is equal to $\frac{i''}{f}$, $i''$ denoting the linear magnitude of the second image. But the angle which the object subtends at the centre of the object-speculum (or to the naked eye, *q. p.*) is equal to that subtended by the first image at the same place, *i. e.*, equal to $\frac{i}{F}$, $i$ denoting the linear magnitude of the first image. And the ratio of these angles, or the *magnifying power*, is

$$M = \frac{F}{f} \times \frac{\mathrm{A}'f}{\mathrm{A}'\mathrm{F}};$$

since $\frac{i''}{i} = \frac{\mathrm{A}'f}{\mathrm{A}'\mathrm{F}}$ (38). But $\mathrm{A}'f$ and $\mathrm{A}'\mathrm{F}$ being conjugate focal distances, $\frac{\mathrm{A}'f}{\mathrm{A}'\mathrm{F}} = \frac{\mathrm{F}'f}{\mathrm{F}'\mathrm{A}'}$, $\mathrm{F}'$ being the principal focus of the small speculum; and since the first image is nearly at the principal focus of the small speculum, and the second nearly at the vertex of the greater, $\mathrm{F}'f = \mathrm{FA}$, *q. p.*, and the preceding ratio is equal to $\frac{F}{F'}$, very nearly. Hence the magnifying power is, approximately,

$$M = \frac{F^2}{F'f}.$$

The second image being inverted with respect to the first, and the first with respect to the object, it is evident that an object seen through this telescope will appear *erect.*

153. The *quantity of light* received by the object-speculum in the Gregorian telescope is to that received by the naked eye from the same object, in the ratio of the difference of the areas of the two specula to that of the pupil; or in the ratio of $A^2 - A'^2$ to $a^2$, $A$ and $A'$ denoting the linear apertures of the two specula, and $a$ that of the pupil. Wherefore, if $p$ denote the ratio of the emergent to the incident light, after reflexion by the two specula and transmission through the eye-glass, the quantity of light received by the eye through the telescope will be

$$p\,\frac{A^2 - A'^2}{a^2};$$

that received by the naked eye being unity. On this quantity depends the *penetrating power* of the instrument.

The *apparent brightness* of an object seen through the telescope varies as the quantity of light in the image on the retina, divided by the space over which it is diffused there. Now the area of the image on the retina is equal to $M^2$, that of the image on the retina of the naked eye being unity; wherefore the brightness of an object seen through the telescope is

$$p\,\frac{A^2 - A'^2}{a^2 M^2};$$

the brightness of the same object to the naked eye being unity.

In order that the brightness of the centre of field should be as great as possible, the small speculum must be of such dimensions as exactly to receive the whole cone of rays converging to the principal focus of the object-speculum. If it be less, it will not reflect the whole of the pencil reflected by the object-speculum; and if it be greater, it will intercept more than is necessary of the parallel pencil incident upon the same.

The hole in the vertex of the object-speculum must not exceed the aperture of the small speculum; and it is evident that, as far as the principal pencil is concerned, no advantage will be gained by making it less. It is usual to make them equal, in order that the aperture of the eye-glass (which is limited by it), and therefore the field of view, may be as great as possible.

154. The angle under which an object is seen in the Gregorian telescope being equal to $\frac{i''}{f}$, that which it subtends to the naked eye is equal to $\frac{i''}{Mf}$, $M$ being the magnifying power. And this latter angle is greatest, and measures the extreme *field of view*, when $i''$, the linear magnitude of the second image, is greatest. But the greatest value of $fg$ (fig. 38) is determined by connecting the corresponding extremities of the small speculum and eye-glass by the line $B'b$; and, since $af$ is small as compared with $A'f$, it is evident that the intercepted portion, $fg$, is equal to $ab$, very nearly. Hence the greatest value of $i''$ is equal to the linear aperture of the eye-glass; and, if the latter be denoted by $a$, the field of view is

$$\frac{a}{Mf}.$$

Instead of a single eye-glass, it is usual to employ, in reflecting telescopes, the Huygenian double eye-piece. The aperture of the latter being greater than can be given to a single eye-glass, the field of view is enlarged; while, at the same time, the errors of form and colour are diminished.

A *diaphragm* is usually introduced at the place of the anterior focus of the eye-glass, the diameter of which is equal to the linear magnitude of the image formed there, so as to prevent all erratic light from reaching the lens. An *eye-stop* is also placed without the eye-glass, at the place of the intersection of the axes of the extreme pencils with the axis of the instrument.

155. Cassegrain's telescope differs from the Gregorian merely in the form and position of the small speculum; this reflector being *convex*, and placed *between* the object-speculum and its principal focus, at a distance from the latter a little less than its own focal length. In this construction, since the rays of each pencil are incident upon the small speculum converging to the points of an image behind it, and within the principal focus, they will, after reflexion, converge to the points of an image in front of the speculum (34, 35). This image, being thrown near the vertex of the object-speculum, is viewed through an eye-glass placed at a distance from it equal to its own focal length; the object-speculum being perforated in the centre, as in the Gregorian telescope.

After what has been said of the Gregorian telescope, it is unnecessary to enter into detail with regard to this construction. The results will differ from those obtained in the preceding articles merely in the sign of $F'$, the focal length of the small speculum.

Objects seen through this instrument appear *inverted;* for the second image is erect with respect to the first, and therefore inverted with respect to the object. For this reason it is unsuitable to terrestrial purposes, unless with an erector eye-piece, an addition seldom made to the reflecting telescope.

The Cassegrainian construction, although little used, appears to possess some very important advantages. It is shorter than a Gregorian of the same power by about double the focal length of the small speculum. And, what is of more importance, the spherical aberrations of the two specula lie in opposite directions, and therefore partially correct each other; whereas, in the Gregorian telescope, the aberration produced by the object-speculum is increased by that of the small reflector. It appears also from some experiments made by Captain Kater, that the Cassegrainian telescope, from whatever cause, surpasses a Gregorian of the same aperture and power, in brightness as well as distinctness.

156. The reflecting telescope erected by the Earl of Rosse, at Parsonstown, is the largest instrument of the kind ever constructed. The object-speculum is six feet in diameter, and five and a half inches in thickness, and weighs upwards of four tons. It is composed of an alloy of copper and tin, in *definite* proportions,—namely, four equivalents of copper to one of tin; or, by weight, 32 to 14·7, nearly. The speculum has been ground and polished by machinery contrived by Lord Rosse for the purpose, by means of which a form of nearly unerring exactness may be given to it. This form is the paraboloid of revolution. Its focal length is fifty-four feet; and it will sustain a magnifying power of 1000, and upwards, in favourable states of the atmosphere.

The tube of this gigantic instrument is fifty feet in length. Its diameter is eight feet in the middle, and seven feet at the extremities. It moves between two walls of massive masonry, in the plane of the meridian; and the highest engineering skill is displayed in the contrivance of the apparatus for its support and movement. Some idea of the scale of the work may be formed from the fact, that the iron castings alone, connected with its framework and support, weigh 150 tons.

This instrument is of the Newtonian construction. The speculum was cast on the 13th of April, 1842; and the whole was completed in the beginning of 1845.

## CHAPTER XII.

### OF THE REFRACTION OF THE ATMOSPHERE.

157. We have seen that when a ray of light passes through any number of refracting media, bounded by parallel planes, its course will be a broken line, consisting of as many right lines as there are media; and that the direction of the ray in the last medium, and therefore the total deviation, will be the same as if the light had been incident directly upon it out of the first (51). Now if the change of refractive power be *continuous*, the intervals of the parallel surfaces will be indefinitely small, and their number indefinitely great; and the polygonal line becomes a curve, the direction of which, in any part of the varying medium, will be the same as if the ray had been incident directly upon it out of the original medium.

The atmosphere which surrounds the earth is a medium of this kind. For since the density of the air decreases continually with the distance from the surface, its refractive power likewise decreases continually; and though the strata of equal density and equal refractive power are, in fact, spherical surfaces concentric with the earth, yet, on account of the small height of the atmosphere compared with the radius of the earth, the deviation from parallelism throughout any space traversed by a ray of light (unless when it proceeds from a star near the horizon) is inconsiderable. Hence, neglecting the curvature of the atmospheric strata, the direction of the ray in the last stratum, where it meets the earth, is independent of the law of variation of density, and the same as if the light had been incident directly upon a homogeneous atmosphere of a density equal to that of the air at the surface.

158. The direction of the ray, before its incidence upon the

atmosphere, marks the *real* position of the object; its direction when it meets the eye, the *apparent* position; and the angle contained by these directions, or the total deviation of the ray, is the *refraction*. The law of its variation is readily determined on the principles above mentioned. For, if we suppose the ray to undergo refraction at the surface of a homogeneous atmosphere of the same density as that at the earth's surface, the angle of refraction will be equal to the *apparent zenith distance* of the object, the surface of the atmosphere and that of the earth being parallel. Wherefore, if this be denoted by $Z$, and the refraction by $R$, the angle of incidence will be $Z + R$, and we have

$$\sin(Z + R) = \mu \sin Z;$$

$\mu$ denoting the refractive index of air of the same density as that at the earth's surface. Or, expanding the first member of this equation, and making $\cos R = 1$, $\sin R = R \sin 1''$, on account of the smallness of the angle $R$,

$$\sin Z + R \sin 1'' \cos Z = \mu \sin Z;$$

$$\therefore R = \frac{\mu - 1}{\sin 1''} \tan Z.$$

Accordingly, the refraction of a celestial object, whose altitude is not very low, varies as the tangent of the apparent zenith distance.

The ray being bent towards the perpendicular to the surface, on entering a denser medium, it is evident that the apparent zenith distance is less than the true, or that the object appears elevated by the effect of refraction.

159. The refractive index of atmospheric air has been determined with great accuracy by the experiments of Biot and Arago. These authors found that, at the temperature of melting ice, and under a pressure measured by a column of mercury whose height was 76 *centimetres* (29·93 English inches), the value of $\mu - 1$ was ·0002943. And dividing by $\sin 1''$, the coefficient of refraction is

$$\frac{\mu - 1}{\sin 1''} = 60''{\cdot}66.$$

The value of $\mu - 1$, however, varies with the density of the air, to which it is proportional; and consequently changes with the *temperature* and *pressure*. At 29·6 inches of pressure, and at the temperature of 50° Fahr., adopted by Dr. Brinkley as the standard, the coefficient becomes $57''\cdot82$.

The refraction in the horizon, in its mean value, is equal to 32 minutes. At very low altitudes, however, the irregularity of refraction is very great. The variations in the state of the atmosphere at the surface of the earth, arising chiefly from sudden changes of temperature, produce variations in the horizontal refraction frequently exceeding its mean value. Lalande mentions that, by the effect of refraction, the mountains of Corsica are occasionally visible from Genoa, a distance upwards of a hundred miles; and the mountains of Wales have been seen, to their very bases, from the Irish coast, at a distance little less.

160. In a medium composed of parallel strata, whose refractive power is continually increasing or diminishing, a ray of light proceeding from the denser into the rarer part of the medium will be continually bent out of its course, and describe a curve which is concave towards the denser medium. Hence, as the angle of incidence upon the strata of equal density is continually increasing, this angle may approach indefinitely to a right angle; and the ray, becoming parallel to these strata, may be bent back into the denser medium, and describe a second branch of the curve precisely similar to the first, the two branches intersecting the same surfaces at equal angles. The effect in such a case, therefore, will be the same as if the ray had suffered reflexion at some stratum of the rarer medium.

The image of an object seen by such rays will be *inverted*. And the object being seen in the direction of the tangent to the curve at the point where it meets the eye, it will appear *elevated* above its true position when the rarer medium is *above* the denser, and, on the contrary, *depressed* when it is *below*, the ray always inclining towards the denser medium.

It is evident, further, that if the eye be situated in the same stratum of the varying medium with the object, the rays proceeding directly from the object to the eye, through the medium of uniform density, may meet there with the curvilinear rays proceeding out of the rarer medium; and that in this manner two images of the object will be visible, one *erect* by direct rays, and the other *inverted* by the rays which are as it were reflected from the rarer medium.

161. In the ordinary states of the atmosphere these phenomena are not exhibited. For, on account of the slow variation of density, the rays proceeding from the denser into the rarer medium have an immense space to traverse before they can reach the eye situated in the same medium with the object, and are, consequently, extinguished in the atmosphere before they reach the eye. When, from any occasional cause, however, the change of density is very rapid, the appearances which we have been describing are plainly seen. It sometimes happens that the temperature of the sea is higher than that of the atmosphere, the former not cooling so rapidly as the latter, and therefore its temperature not being subject to the same sudden vicissitudes. When this is the case, the superficial stratum of air becomes warmer, and therefore also rarer, than that immediately above it; and the density will increase rapidly up to a certain height, and then diminish gradually. Now if the eye of a spectator be situated in the stratum of greatest density, and be directed to an object in the same stratum, he will observe two images: one erect, by the rays which are transmitted directly through the uniform medium; and the other inverted, by the rays which are, as it were, reflected upwards from the rarer medium. This second image will be depressed.

This phenomenon is frequently observed off the coast of Sicily, where it is attributed by the peasants to a supernatural agent, *Fata Morgana.* The French call it *mirage;* it is known in this country by the name of *looming.*

The same phenomenon, attended with some other remarkable appearances, is observed almost every day on the sandy plains of Lower Egypt; the dry soil of which becomes intensely heated by the midday sun, and thus rarefies the contiguous stratum of air much more than that above it. The plains at a distance become invisible, the light proceeding from objects situated there never reaching the eye. At the same time the villages on the heights, which are above the rarefied air, are seen directly through air of nearly uniform density, together with inverted images below them reflected from the rarer medium. Thus the plain appears like a vast sea, above which the villages are seen like islands, together with their images beneath them reflected as it were from the surface of the water. As the traveller advances, the nearer parts of the plain become visible by the light transmitted thence directly to the eye; and thus the inundation seems to recede, as if in mockery of his thirst.

Appearances similar to these may be produced artificially, by heating a bar of metal, and looking along its edge at a distant object. The upper image, seen through a denser stratum of air, will be erect; while the lower, which is seen by light reflected from the rarer stratum, is inverted.

When the density of the strata *decreases* rapidly from the surface upwards,—as will sometimes be the case when the temperature of the air over the sea is suddenly raised by the heat of the sun,—the effects will be the opposite to those above described, the inverted image being in this case *elevated.*

162. The phenomenon of the *rainbow* is produced by the reflexion and refraction of the sun's rays in the drops of falling rain, the interior or *primary* bow being formed by two refractions and *one* intermediate reflexion, and the exterior or *secondary* bow by two refractions and *two* reflexions.

When a parallel pencil of the sun's rays is incident upon a spherical drop of rain, a portion of each ray enters the drop in an altered direction, and thus meets its surface again internally. Part of this ray will emerge, and part be reflected, so as to meet the surface a second time. It will there undergo a new subdivision, part emerging from the drop, and part being reflected a second time internally; and so on. Now the rays which compose the pencil being incident upon the sphere at every possible angle from 0° to 90°, the rays which emerge after any given number of internal reflexions will be inclined to the incident rays, and therefore to one another, at every possible angle within certain limits. It may be shown, however, that there is a particular angle of incidence for which the emergent rays will be nearly *pa-*

*rallel;* so that the eye will receive a greater number of emergent rays, and therefore be more strongly affected, when placed in their direction, than in any other. The angle of incidence at which this takes place being given, the angle formed by the incident and emergent rays is likewise given.

Let O (fig. 40) be the place of the eye of a spectator, having his back to the sun; and SOS′ the line drawn through O to the sun's centre. This line is sometimes called the *axis of vision*. Let S′O$a$ be the angle formed by the incident and emergent rays, when the rays emerge parallel after *one* internal reflexion; and S′O$a'$ the corresponding angle when they emerge parallel after *two* internal reflexions. Then it is evident that if, with the line OS′ as an axis, two conical surfaces be described with these angles, every straight line forming a side of these cones will be inclined to the incident light at the required angle; and, consequently, all the drops situated in these surfaces will transmit parallel rays to the eye at the vertex. Hence, two circular arches will be seen, in which the light is more intense than in any other direction. The radii of these arches are the angles S′O$a$, S′O$a'$; and as these angles vary with the refractive index, and are therefore different for the different species of simple light, the arches will be coloured, the order of colours being the same as in the spectrum.

The *altitude* of any arc of the rainbow, HO$a$, is equal to the radius of that arc, S′O$a$, diminished by the altitude of the sun SOZ. Hence the bow is invisible, when the altitude of the sun is equal to, or greater than, the radius. The altitude of the bow is greatest when the sun is in the horizon. The *breadth* of the bow is equal to the variation of the angle S′O$a$, corresponding to the extreme variation of the refractive index of water.

Since a portion of light emerges from the drop at each reflexion, the light which reaches the eye is fainter the greater the number of reflexions. Hence the secondary bow is much more faint than the primary; and the *tertiary* bow is seldom seen.

163. It is required to determine the course of a ray of light which enters a sphere, and emerges after any number of internal reflexions.

It is evident that the ray will continue in the same plane throughout, namely, the plane containing the incident ray and the centre of the sphere. For the refracted ray, both at incidence and emergence, is in the plane containing the incident ray and the radius at the point of incidence; and each reflected ray is in the plane containing the incident ray and the radius drawn to point of incidence.

Let RSTV (fig. 39) be the great circle containing the incident ray, and QRSTVX the course of the ray which is incident and emergent at R and V, and reflected at S, T, &c. Then it is evident that the angles contained by each chord with the radii drawn to its extremities, CRS and CSR, CST and CTS, &c., are respectively equal; as are also the angles contained by each radius with the adjacent chords, CSR and CST, CTS and CTV, &c., these being the angles of incidence and reflexion. Hence all the angles contained by the direction of the ray within the sphere with radius are equal; wherefore the angles CRS and CVT at incidence and emergence are equal, and accordingly the ray emerges under an angle equal to that of incidence.

If $\phi$ and $\psi$ denote the angles of incidence and refraction into the sphere, the angle of deviation at the point of incidence is $\phi - \psi$; and, as the deviation is the same at emergence, the sum of the deviations at incidence and emergence is $2(\phi - \psi)$. Again, the deviation at each point of reflexion is equal to $\pi - 2\psi$, each angle of reflexion being equal to the angle of refraction into the sphere. Wherefore, if the number of reflexions which the ray undergoes within the sphere be denoted by $n$, the sum of the deviations at the points of reflexion will be $n\pi - 2n\psi$. Accordingly, the total deviation is

$$n\pi + 2\phi - 2(n+1)\psi.$$

164. Required the angle of incidence, for which the emergent rays are nearly parallel.

The emergent rays being parallel, as well as the incident, the angle contained by the incident and emergent rays (and therefore the deviation) does not vary with small changes of incidence; in other words, the differential of the angle of deviation = 0. Hence there is

$$d\phi = (n+1)d\psi.$$

But we have

$$\sin\phi = \mu \sin\psi, \qquad \cos\phi\, d\phi = \mu \cos\psi\, d\psi;$$

and, substituting in the latter the value of $d\phi$ just obtained,

$$(n+1)\cos\phi = \mu \cos\psi;$$

which equation, combined with the former, will determine the angle of incidence required. To effect the elimination, we have only to square these equations and add them together, and we find

$$(n+1)^2\cos^2\phi + \sin^2\phi = \mu^2;$$

$$\text{or, } \left[(n+1)^2 - 1\right]\cos^2\phi = \mu^2 - 1;$$

$$\therefore \cos\phi = \sqrt{\frac{\mu^2-1}{n(n+2)}}.$$

165. It is required to determine the radius of the primary bow.

Making $n = 1$, in the formula of (163), the deviation is equal to $\pi + 2\phi - 4\psi$; and consequently the angle formed by the incident and emergent rays, or $\pi$ - *angle of deviation*, is equal to

$$4\psi - 2\phi,$$

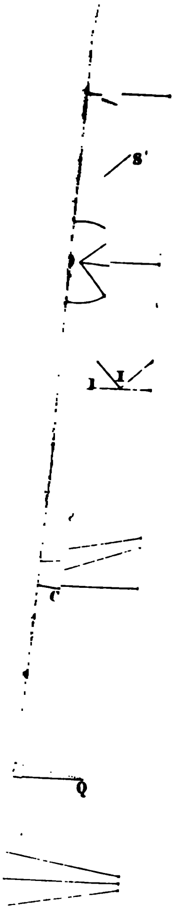
and, making $n = 1$ in the formula of (164),

$$\cos\phi = \sqrt{\frac{\mu^2-1}{3}}.$$

$\phi$ being found, $\psi$ is deduced from the equation $\sin\psi = \frac{1}{\mu}\sin\phi$; and, the values of these angles being substituted in the expression above given, the angle formed by the incident and emergent rays, or the radius of the bow, is obtained.

The value of this angle obviously depends upon that of the refractive index, and is, therefore, different for each of the different species of simple light. Now the values of the refractive index of water, for the *red* and *violet* rays respectively, are 1·333 and 1·346. And, calculating with these values of $\mu$, the values of the radius are found to be, respectively,

42° 2′ and 40° 17′.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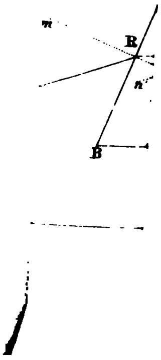
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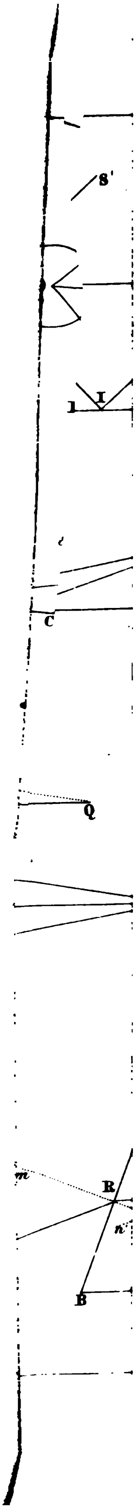
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